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BUCKLING ANALYSIS OF RECTANGULAR ORTHOTROPIC PLATE SUBJECTED TO IN-PLANE BENDING

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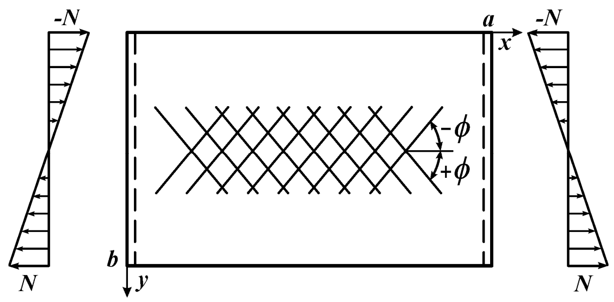
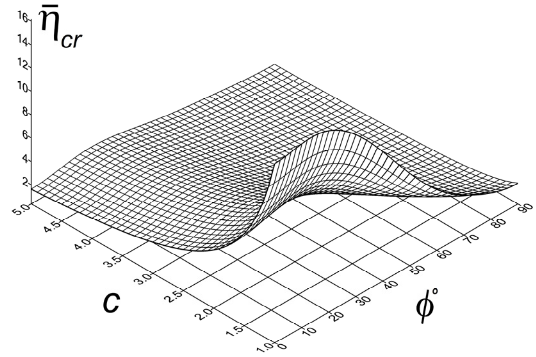
ABSTRACT

The buckling of a rectangular plate loaded along the two parallel simply-supported opposite edges by the linearly distributed in-plane load is considered in this paper (see Fig.1 (a)). Under the action of this load the plate is subjected to the in-plane pure bending. The corresponding buckling solutions based on the use of double trigonometric series were obtained for simply supported isotropic [1-2] and orthotropic [3-5] plates. It appears, to the authors' knowledge, that no research work was reported for the buckling analysis of the plate under consideration having two parallel edges simply supported with no loads applied on the remaining free edges (SSFF plate).



(*a*)



(*b*)

Fig.1. Plate under in-plane pure bending (a), coefficient, $\overline{\eta}_{cr}$, as a function of angle ϕ and aspect ratio c for composite plate (b).

The buckling equation for the orthotropic $a \times b$ plate referred to the Cartesian coordinate frame (x, y) (see Fig.1) and loaded along the two parallel simply supported opposite edges($x = 0, a$) by the linearly distributed in-plane load $N_x^0 = -N(1-2y/b)$ is:

$$D_{11}\,\partial^4 w/\partial x^4 + 2(D_{12} + 2D_{33})\,\partial^4 w/\partial x^2 \partial y^2 + D_{22}\,\partial^4 w/\partial y^4 + N(1-2y/b)\partial^2 w/\partial x^2 = 0 \quad (1)$$

where $w = w(x, y)$ is the lateral deflection, D_{11}, D_{12}, D_{22}, D_{33} are the flexural stiffnesses of the laminated plate [6], N is the maximum value of the pre-buckling distributed load.

The boundary conditions are as follows: $w = 0$, $D_{11}\partial^2 w/\partial x^2 + D_{12}\partial^2 w/\partial y^2 = 0$ for $x = 0, a$, and $D_{12}\partial^2 w/\partial x^2 + D_{22}\partial^2 w/\partial y^2 = 0$, $D_{22}\partial^3 w/dy^3 + (D_{12} + 4D_{33})\partial^3 w/\partial x^2\partial y = 0$ for $y = 0, b$. Using the Levy solution form which satisfies the boundary conditions for simply supported edges $x = 0, a$, the equation (1) is reduced to the ordinary differential equation. The latter is approximated by the finite differences as follows:

$$\alpha\pi^2 w_i - 2\beta n^2 A_i + B_i n^4 \big/ \alpha\pi^2 - \eta t_i w_i = 0 \qquad (2)$$

where i is the mesh point, w_i is the plate deflection at the point i, $A_i = w_{i-1} - 2w_i + w_{i+1}$, $B_i = w_{i-2} - 4w_{i-1} + 6w_i - 4w_{i+1} + w_{i+2}$, $t_i = 1 - 2(i-1)/n$, $\beta = (D_{12} + 2D_{33})/\sqrt{(D_{11}D_{22})}$, $\alpha = \sqrt{(D_{11}/D_{22})}b^2/a^2$, $i = 1,2,...,k$, $k = n+1$, n is the number of partitions of the edge [0, b] and $\eta = Nb^2\big/\sqrt{D_{11}D_{22}}$ is the buckling coefficient. Applying Eq. (2) to all the mesh points and taking into account the finite difference approximations of the boundary conditions, we obtain the following set of equations

$$(\mathbf{D} - \eta\mathbf{T})\mathbf{W} = 0 \qquad (3)$$

where, $\mathbf{W} = \{w_1, w_2, ..., w_k\}^T$, $\mathbf{D} = \alpha\pi^2\mathbf{E} - 2\beta n^2\mathbf{A} + n^4\mathbf{B}\big/\alpha\pi^2$, $\mathbf{T}$ is the diagonal matrix with the main diagonal elements $t_1,...,t_k$, $\mathbf{E}$ is the unity matrix, and $\mathbf{A}$ and $\mathbf{B}$ are the square tridiagonal and band diagonal matrices of the numerical coefficients in the expressions for A_i and B_i respectively. Solution of the eigenvalue problem (3) yields the critical buckling coefficient η_{cr} with the corresponding critical load $N_{cr} = \eta_{cr}\sqrt{D_{11}D_{22}}/b^2$. The problem was solved for the isotropic plate and the laminated CFRP (carbon fiber reinforced plastic) plate composed from the orthotropic layers with symmetrical reinforcement orientation $\pm\phi$. Coefficient $\overline{\eta}_{cr} = \eta_{cr}\sqrt{A_{11}A_{22}}/E_1$ (A_{11}, A_{22} are the stiffness coefficients and E_1 is the modulus elasticity of unidirectional composite [6]) was calculated for various combinations of the reinforcement angle $0^\circ \le \phi \le 90^\circ$ and the aspect ratio $1 \le c \le 5$ ($c = a/b$) (see Fig. 1 (b)). It was found, that for each value of c there is a corresponding angle $\phi = \hat{\phi}$ that delivers the maximum value of the critical buckling coefficient. For example, $\hat{\phi} = 14^\circ$ for the square ($c = 1$) CFRP plate. It has been shown, that the optimum angle of reinforcement (corresponding to the maximum critical load) tends to the value of 22° with the increase of the aspect ratio c. The effect of the geometric and elastic parameters of the plates on the shape of corresponding buckling modes was also analysed.

REFERENCES

[1] I.G. Bubnov, *Theory of ship structures*, Vol. I and II, St. Petersburg, 1912, 1914.

[2] S.P. Timoshenko and J.M. Gere, *Theory of elastic stability*, 2nd Edition, McGraw-Hill, N.Y., 1961.

[3] S.G. Lekhnitskii, *Anisotropic plates*, Gordon & Breach, N.Y., 1968.

[4] J.N. Reddy, *Theory and analysis of elastic plates*, Taylor & Francis, 1998.

[5] J.M. Whitney, *Structural analysis of laminated anisotropic plates*, Technomic Publ., Co., Lancaster, Pennsylvania, 1987.

[6] V.V. Vasiliev, *Mechanics of composite structures*, Taylor & Francis, 1993.