8th. World Congress on Computational Mechanics (WCCM8)
5th European Congress on Computational Methods in Applied Sciences and Engineering (ECCOMAS 2008)
June 30 –July 5, 2008
Venice, Italy

A high order accurate finite difference method for adaptive grids

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Key Words: *Non-overlapping grids, Adaptive grids, Fluid Dynamics, High order finite difference.*

ABSTRACT

High-order accurate finite difference methods (HOFDM) have been proven [1] highly efficient for Cauchy problems. The usage of HOFDM for more realistic problems have been delayed by two major obstacles: 1) To device a stable boundary treatment, and 2) To handle complex domains.

The first obstacle was resolved by the combination of high order accurate summation by parts (SBP) operators [2,3] and the Simultaneous Approximation Term (SAT) method [4] for implementing the physical boundary conditions. The second obstacle was partly resolved [5] by employing the SAT technique to couple problems solved on multi-block curvilinear grids, that were allowed to have a non smooth matching of gridlines.

We will present newly derived high order accurate interpolation operators that allow us to couple problems on non-matching curvilinear multi-block grids, that are allowed to have a two to one ratio of unknowns at the interface. This will allow us to construct adaptively refined meshes, to resolve complex geometrical features more efficiently. The strength of this technique is that we can prove that the interface coupling is stable and conservative, and preserves design order of accuracy.

As a validation test we consider a 2-block coupling of the 2-D compressible Euler equations. The analytic Euler vortex [6] is imposed as initial data close to the internal boundary at x=5, see Figure 1 . The vortex propagate with the free stream speed to the right. The solution (here density) at $t = 1$, using 81^2 and 41^2 grid points in the left and right domains respectively is shown in Figure 1. We use a 4th order accurate SBP operator (having a 6th order accurate interior stencil and a 3rd order accurate boundary closure) and the standard 4th order accurate Runge-Kutta method to integrate in time. We stress that the interface coupling technique is not restricted to any particular PDE, as will be illustrated at the conference.

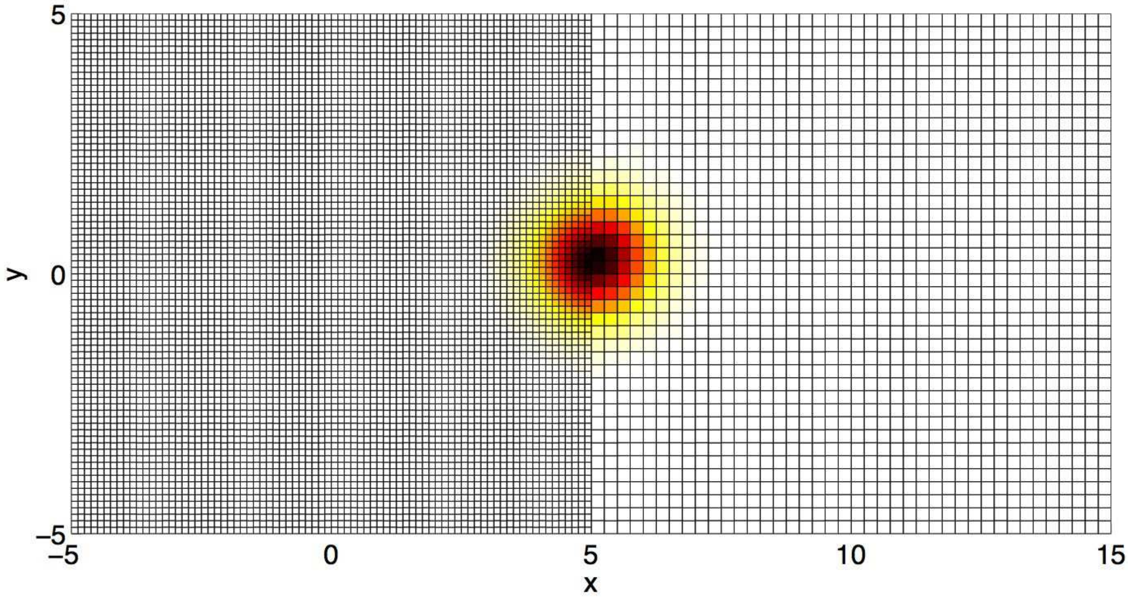

Figure 1: The vortex (density), at $t = 1$. The left (fine) block uses 81^2 grid points. $Ma = 0.7$.

N	$log(\rho)$	$q^{(\rho)}$	$log(l_2(u))$	$q^{(u)}$	$log(l_2(v))$	$q^{(v)}$	$log(l_2(E))$	$q^{(E)}$
101^2	-5.73	0.00	-5.27	0.00	-4.54	0.00	-4.28	0.00
201^2	-6.96	4.10	-6.23	3.21	-5.92	4.57	-5.58	4.35
301^2	-7.79	4.71	-6.94	4.04	-6.76	4.76	-6.43	4.82
401^2	-8.38	4.75	-7.48	4.30	-7.38	4.98	-7.03	4.76

Table 1: l_2-errors and convergence rates q for the unknowns (density, velocities and energy). The errors are sampled at $t = 1$. N denote the number of grid points in the left (fine-grid) domain. $Ma = 0.3$.

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