8th. World Congress on Computational Mechanics (WCCM8)
5th European Congress on Computational Methods in Applied Sciences and Engineeering (ECCOMAS 2008)
June 30 –July 5, 2008
Venice, Italy

ON THE CONSTRUCTION OF GENERALIZED SPECTRAL BASES FOR SOLVING STOCHASTIC PARTIAL DIFFERENTIAL EQUATIONS

Anthony Nouy

Research Institute in Civil Engineering and Mechanics (GeM)
Nantes University, Ecole Centrale Nantes, UMR CNRS 6183
2 rue de la Houssinière, B.P. 92208, 44322 Nantes Cedex 3, FRANCE
anthony.nouy@univ-nantes.fr

Key Words: *Stochastic partial differential equations, Stochastic Finite Element, Generalized Spectral Decomposition, Invariant Subspace problem, Stochastic Model Reduction.*

ABSTRACT

The introduction of uncertainties inherent to physical models is a key question when one tries to obtain reliable numerical predictions. Stochastic Galerkin methods [1,2,3] have become a significant tool for solving stochastic partial differential equations (SPDE). These methods are based on strong mathematical bases, which allow deriving *a priori* and *a posteriori* error estimators. They lead to very accurate and reliable approximate solutions. However, when a fine discretization is required at deterministic and/or stochastic levels, these methods need for the resolution of huge systems of equations. The use of classical solvers then generally induces very high computational costs.

A rising tendency in computational stochastic mechanics consists in building low dimensional approximation spaces in order to drastically reduce the size of discretized problems. A reduced basis of deterministic functions $\{U_i\}$ or random variables$\{\lambda_i\}$ is said optimal if, for a given accuracy, it leads to the lowest order M of decomposition of the solution $u \approx \sum_{i=1}^{M} \lambda_i U_i$. Optimality clearly depends on the "measure of accuracy". For example, when measuring accuracy with the natural norm in a tensor product Hilbert space, the optimal decomposition appears to be the Karhunen-Loeve decomposition (or classical spectral decomposition) of the solution. Of course, this decomposition can not be obtained without knowing the solution. However, different numerical strategies have been proposed in order to obtain an approximation of it [2,4]. This approximate decomposition allows the obtention of quasi-optimal reduced deterministic (or stochastic) bases on which the initial fine problem can be solved.

Recently, a new method has been proposed for building an optimal decomposition without *a priori* knowing the solution nor an approximation of it [5]. This method, called Generalized Spectral Decomposition method (GSD), starts with another definition of optimality and require the development of ad-hoc algorithms for the automatic construction of the decomposition.

The GSD method has been initially derived for a class of elliptic SPDE [5]. In this presentation, the GSD method is extended to a wider class of stochastic problems. The GSD is defined by classical Galerkin orthogonality criteria. Reduced bases are solutions of invariant subspace problems (or fixed point problems on Grassmann manifolds [6]), which are interpreted as eigen-like problems. This interpretation allows to derive new efficient algorithms to build the generalized spectral decomposition.

These are inspired from classical algorithms for capturing dominant eigenspaces of linear operators. The proposed algorithms leads to significant computational savings by separating the resolution of a few deterministic problems and a few stochastic problems on reduced deterministic bases.

The proposed strategy offers a quite general framework for solving a large class of stochastic partial differential equations. Numerical examples illustrate the generality of the method and the efficiency of the proposed algorithms, which are compared to classical resolution techniques and also to the previous GSD algorithms proposed in [5].

REFERENCES

[1] R. Ghanem and P. Spanos. *Stochastic finite elements: a spectral approach.* Springer, Berlin, 1991.

[2] H. G. Matthies and A. Keese. "Galerkin methods for linear and nonlinear elliptic stochastic partial differential equations". *Computer Methods in Applied Mechanics and Engineering*, Vol. **194**(12-16),1295–1331, 2005.

[3] I. Babuška, R. Tempone, and G. E. Zouraris. "Solving elliptic boundary value problems with uncertain coefficients by the finite element method: the stochastic formulation". *Computer Methods in Applied Mechanics and Engineering*, Vol. **194**, 1251–1294, 2005.

[4] A. Doostan, R. Ghanem, and J. Red-Horse. "Stochastic model reductions for chaos representations". *Computer Methods in Applied Mechanics and Engineering*, Vol. **196**(37-40), 3951–3966, 2007.

[5] A. Nouy. "A generalized spectral decomposition technique to solve a class of linear stochastic partial differential equations". *Computer Methods in Applied Methods in Engineering*, Vol. **196**(45-48), 4521–4537, 2007.

[6] A. Edelman, T. A. Arias, and S. T. Smith. "The geometry of algorithms with orthogonality constraints". *SIAM J. Matrix Anal. Appl.*, Vol. **20**(2), 303-353, 1998.