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HERMITE REPRODUCING KERNEL MESHFREE ANALYSIS OF THIN BEAMS AND PLATES

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ABSTRACT

The non-polynomial characteristic of typical meshfree shape functions like moving least square (MLS) or reproducing kernel (RK) approximation makes it necessary using higher order Gauss integration (GI) to accurately evaluate the strain energy in elasticity problems where the 1st order derivatives of shape functions are involved. This issue becomes more prominent when dealing with the problems of thin beams and plates where the 2nd order derivatives of shape function are required to formulate the stiffness. To improve the computational efficiency methods have been proposed by using stabilized nodal integration. In [1] a least squares stabilization was proposed to the nodally integrated weak form and in [2] the stabilized conforming nodal integration (SCNI) was developed via a strain smoothing method. The SCNI approach has been further extended to shear deformable beams, plates and shells [3-5]. It is noted that the SCNI type stabilization methods were formulated to meet the so-called integration constraints in elasticity and plate bending, and thus pass linear and pure bending tests. However the aforementioned stabilization methods were developed for the 2nd order differential equations. This work aims to present a Hermite reproducing kernel (HRK) meshfree approximation and a corresponding stabilization method for in higher order differential equations such as thin beams and plates.

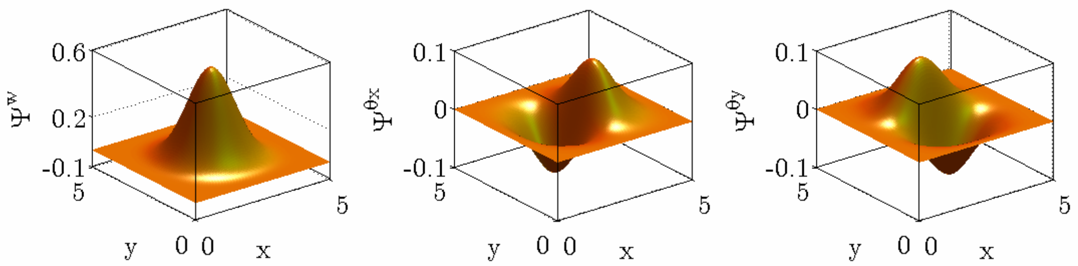

Figure 1 Hermite reproducing kernel shape functions for deflection and rotations

Under the present approach, the HRK approximation is constructed by including the rotations into the approximation of deflection and then imposing the consistency conditions [6]. In this approximation the Kirchhoff mode reproducing conditions are enforced to exactly reproduce the fundamental plate bending deformation modes. It turns out that the proposed HRK shape functions as shown in Fig. 1 exhibit a better kernel stability compared to that of the standard reproducing kernel shape functions. It is also shown that the HRK approximation has better approximation accuracy than that based on the standard RK approximation. Since the weak form of thin plates involves the 2nd order derivatives, accurate numerical integration of the discrete stiffness matrix requires very intensive computational effort. Numerical tests also demonstrate that a direct employment of previous SCNI methodology or lower order Gauss integration rules are not enough to yield a stable meshfree algorithm for the solutions of thin beams and plates. Consequently a sub-domain stabilized conforming integration (SSCI) is developed to efficiently and accurately integrate the weak forms of thin beams and plates, and meet the stability requirement at the same time. Due to its conforming nature and the use of strain smoothing technique, the proposed formulation with HRK approximation and SSCI integration scheme can meet the integration constraint associated with bending and thus reproduce pure bending mode under arbitrary particle discretization patterns. Various benchmark examples, i.e., the clamped square plate problem as shown from Fig. 2, demonstrate that the proposed method performs superiorly compared to the higher order Gauss integration-based meshfree formulation.

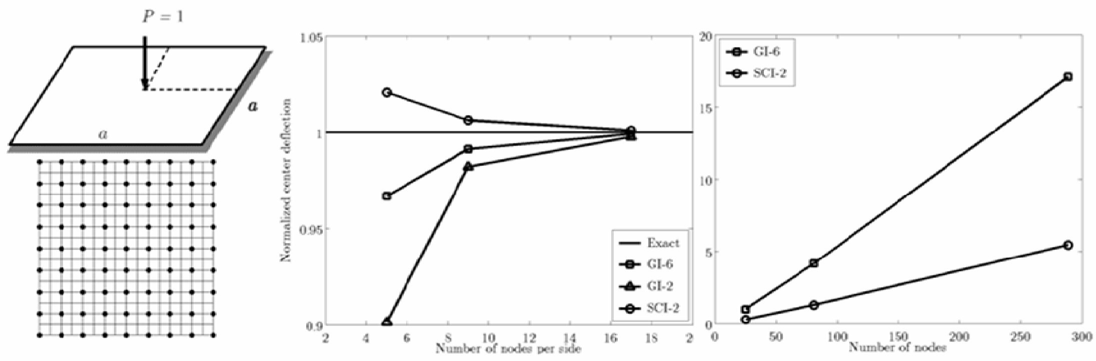

Figure 2 Solution comparisons for the clamped square plate under center load

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