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Transonic Viscous Inviscid Interactions of Dense Gases in Narrow Channels

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ABSTRACT

Steady transonic flows through channels so narrow that the classical boundary layer approach fails are considered. As a consequence the properties of the inviscid core and the viscosity dominated boundary layer regions adjacent to the channel walls can no longer be determined in subsequent steps but have to be calculated simultaneously, thus allowing viscous inviscid interactions to take place locally which are triggered, e.g., by the formation of a weak normal shock. Under the requirement that the channel is so narrow that the flow outside the viscous wall layers becomes one-dimensional to the leading order the resulting interaction problem is formulated, by using asymptotic analysis for large Reynolds number $Re = \tilde{u}_{-\infty}\tilde{L}_{-\infty}/\tilde{\nu}_{-\infty} \to \infty$, for single-phase fluids with either positive, negative or mixed nonlinearity, that is to say, the fundamental derivative of thermodynamics $\Gamma = (1/\tilde{c})\partial(\tilde{\rho}\tilde{c})/\partial\tilde{\rho}|_{\tilde{s}}$ is strictly positive, negative or changes sign, respectively, in the region of interaction. Here $\tilde{c}, \tilde{\rho}, \tilde{s}, \tilde{u}_{-\infty}, \tilde{L}_{-\infty}, \tilde{\nu}_{-\infty}$ are the local speed of sound, the density, the entropy, and reference values for the flow velocity, for a characteristic length and for the kinematic viscosity associated with the unperturbed boundary layers, fig. 1, a tilde denotes dimensional quantities. A prominent example for fluids with mixed nonlinearity are dense gases with relatively large specific heats, also referred to as BZT fluids, see [1].

The interaction region exhibits a triple deck structure, fig. 1, where, as in classical triple deck theory, e.g. see [3], the role of the main deck is to transfer the displacement effects exerted by the lower deck (LD) unchanged to the upper deck (UD) and the resulting pressure disturbances again unchanged to the LD. Here, the fluid motion is governed by the boundary layer equations in incompressible form supplemented by the usual no slip condition at the wall and new matching conditions, [3]. The displacement effect caused by the LD is quantified by the perturbation of the displacement thickness $-A(X)$, with X a local suitable scaled coordinate in streamwise direction, see fig. 1, weakly perturbing the quasi one-dimensional isentropic UD flow governed by the relation

$$G_{(n)}\left(P(X); K_{-\infty}, \Gamma_{-\infty}, \Lambda_{-\infty}, N_{-\infty}\right) = QA(X) \tag{1}$$

where P denotes the pressure disturbance and Q is associated with the strength of the coupling between LD and UD deck flow, all quantities are suitable scaled. $G_{(n)}$ is a polynomial function of the pressure of order $n \in \{2, 3, 4\}$ and, following [2], can be interpreted as a local perturbation of the mass flux density in the inviscid core region with the property that an extremum of $G_{(n)}(P)$ corresponds to a sonic state in the core region flow. Consequently, the case $n = 2$ describes the behaviour of fluids that

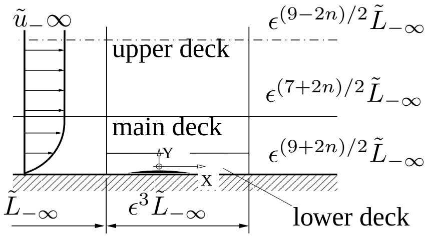

Figure 1: Triple deck structure of interaction region; $\epsilon := Re^{-1/(7+2n)}$.

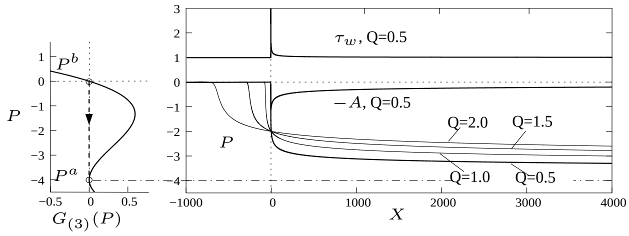

Figure 2: Regularised sonic rarefaction shock; PP10: $\Gamma_{-\infty}$ = -0.051, $\Lambda_{-\infty}$ = -0.92; τ_w . . . wall shear stress.

pass through a sonic state only once during isentropic expansion in the considered flow region, which is the case for fluids with strictly positive or negative nonlinearity and thus can be described locally by $\Gamma_{-\infty}$, where the index $-\infty$ has the meaning evaluated at upstream flow conditions. The case $n = 3$ and $n = 4$ describes the situation for fluids with mixed nonlinearity where sonic conditions can be reached two or three times, respectively, and thus $\Lambda_{-\infty} := \partial\Gamma/\partial\rho|_{s,-\infty}$ and $N_{-\infty} := \partial^2\Gamma/\partial\rho^2|_{s,-\infty}$ have to be considered too. We take the transonic similarity parameter $K_{-\infty} > 0$ for subsonic and $K_{-\infty} < 0$ for supersonic upstream flow conditions.
Nontrivial eigensolutions of the considered interaction problem connecting unperturbed up- and downstream flow conditions, $A(X) \to 0$ for $X \to \pm\infty$, correspond to the internal structures of weak normal shocks, where the values of P before and after the shock, P^b and P^a, have to satisfy the jump condition $G_{(n)}(P^a) = G_{(n)}(P^b)$ expressing the continuity of the mass flux across a shock front, [2]. The solutions for the internal structure of weak rarefaction, sonic, double sonic and split shocks that will be presented are therefore regularised by the mechanism of viscous inviscid interaction completely different to that of thermoviscosity found in literature, see e.g. [2]. However the results obtained are, as will be shown, equally in accordance with the admissibility criteria formulated in [2] for a nonconvex flux function. In the following instructive solutions for the internal structure of sonic rarefaction shocks with $M^b > 1$ and $M^a = 1$, where M denotes the local Mach number, for different values of the parameter Q are briefly discussed for the example medium PP10 ($C_{13}F_{22}$), [1]. The thermodynamic properties of PP10 are calculated according to the Martin-Hou EOS using the material parameters published in [1]. Unlike to compression shocks the flow in the boundary layer is accelerated reducing the displacement thickness $-A(X)$, fig. 2, thus avoiding the risk of flow separation frequently resulting from shock/boundary layer interaction. For weaker viscous inviscid interactions, i.e. smaller values of Q, the pressure distribution in fig. 2 is more and more approaching the shock solution predicted by inviscid theory.
Besides the theoretical value of the discussion presented, these solutions are expected to give a qualitative insight into flow phenomena to be expected in the flow of BZT fluids through slender channels or Laval nozzles, as BZT fluids could prove beneficial as working fluids in ORC processes in the future. Furthermore the set up described here could pose an alternative to the attempt of using a shock tube for the experimental detection of rarefaction shocks, [1], with the distinguishing property that the shock position is stationary and that no other shock or wave phenomena would have to be accounted for.

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