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ABSTRACT TITLE

Well-balanced high order methods for Balance Laws based on reconstruction of states

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ABSTRACT

This paper is concerned with the numerical approximation of Cauchy problems for one-dimensional balance laws:

$$w_t + F(w)_x = G(w)\sigma_x, \quad x \in \mathbf{R},\ t > 0, \tag{1}$$

where the unknown $w(x,t)$ takes values on an open convex set $\mathcal{O}$ of $\mathbf{R}^N$; σ is known function from $\mathbf{R}$ to $\mathbf{R}$; and F, G are regular functions from $\mathcal{O}$ to $\mathbf{R}^N$. The shallow water system with source terms due to bed elevations can be formulated in this form.

Some numerical difficulties arise when solving steady-state solutions or solutions close to equilibrium. In this case, if the numerical source term doesn't balance the flux term in an adequate manner, the numerical solutions may exhibit non-physical oscillations. The development and the analysis of well-balanced numerical schemes for systems of balance laws is a very active front of research and there is a huge recent literature devoted to this topic. We refer to [1], [5], and the references therein for a review on the subject.

The goal of this paper is to develop high order well-balanced numerical schemes for systems of the form (1). To do this, σ is first interpreted as an artificial unknown of the system, whose value is fixed by the initial condition. Then, (1) is rewritten under the form:

$$W_t + A(W) \cdot W_x = 0, \quad x \in \mathbf{R},\ t > 0, \tag{2}$$

where $W = [w, \sigma]^T$. Once the system has been rewritten in this form, a well-balanced first order path-conservative numerical scheme is considered (see [4]). For instance, a numerical scheme based on an approximate Riemann solver can be chosen. Next, we consider a higher order extension of the first order numerical scheme based on the use of a reconstruction operator.

Standard reconstruction operators (as ENO, WENO, etc.) do not preserve in general the well-balance property of the first order scheme. Here we present a technique introduced in [2] to modify a standard reconstruction operator to solve this difficulty. The idea is to use, in the reconstruction procedure, the information given by the integral curves of the linearly degenerate field associated to the null eigenvalue of the matrix $A(W)$. These curves, as it will be seen, are related both to the stationary contact discontinuities and to the regular stationary solutions of (2).

In the talk, this methodology will be applied to general semi-linear balance laws and to the shallow water system. The differences and similarities of this approach with the one pursued in [3] to obtain well-balanced high order schemes will be also briefly discussed.

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