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CONGESTED TRAFFIC SIMULATION BASED ON A 2D HYDRODYNAMICAL MODEL

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ABSTRACT

The problem of modeling vehicular traffic flows on city roads and at freeways is considered. Now there are two basic concepts of traffic flow modeling [1]. In so-called “microscopic” models cars are considered as individual particles which interact under the certain laws. Other type of models is “macroscopic” or hydrodynamical, where the traffic is considered as a compressible liquid. There are also kinetic (mesoscopic) models representing an intermediate step between the above two kinds of models.

Most of the existent models are one-dimensional. The present work is devoted to further development of an original 2D macroscopic model of multilane vehicular traffic [2] to predict flows for real road geometry. The case of congested traffic is under consideration: distances between cars are comparable with car sizes and velocities are far from the free flow velocity. Under these conditions on long intervals of a road it is possible to use the approach of a continuous medium and derive equations like gas dynamics equations. Notions of the density $\rho(x, y, t)$ as a quantity of vehicles per lane in a distance unit and the flux as a function of the density and the velocity are employed in the traditional way.

The model is constructed by the analogy with the Quasi-Gas-Dynamic System of Equations [3]. Additional mass fluxes guarantee smoothing the solution on the reference distances. The system of traffic flow equations is as follows:

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u}{\partial x} + \frac{\partial \rho v}{\partial y} = \frac{\partial}{\partial x}\left(\frac{\tau_x}{2}\frac{\partial}{\partial x}\left(\rho u^2 + P\right)\right) + \frac{\partial}{\partial x}\left(\frac{\tau_x}{2}\frac{\partial}{\partial y}\left(\rho u v\right)\right) + \\ + \frac{\partial}{\partial y}\left(\frac{\tau_y}{2}\left(\frac{\partial}{\partial y}\left(\rho v^2\right) + \alpha_y \rho^{\beta_y}\frac{\partial \rho}{\partial y}\right)\right) + \frac{\partial}{\partial y}\left(\frac{\tau_y}{2}\frac{\partial}{\partial x}\left(\rho u v\right)\right) - \frac{\partial}{\partial x}\left(\frac{\tau_x}{2} f\right)$$

$$\frac{\partial \rho u}{\partial t} + \frac{\partial \rho u^2}{\partial x} + \frac{\partial \rho u v}{\partial y} = f - \frac{\partial}{\partial x}P + \frac{\partial}{\partial x}\left(\frac{\tau_x}{2}\frac{\partial}{\partial x}\left(\rho u^3 + 3Pu\right)\right) + \frac{\partial}{\partial x}\left(\frac{\tau_x}{2}\frac{\partial}{\partial y}\left(\rho u^2 v\right)\right) + \\ + \frac{\partial}{\partial y}\left(\frac{\tau_y}{2}\left(\frac{\partial}{\partial y}\left(\rho u v^2\right) + \alpha_y \rho^{\beta_y}\frac{\partial \rho u}{\partial y}\right)\right) + \frac{\partial}{\partial y}\left(\frac{\tau_y}{2}\frac{\partial}{\partial x}\left(\rho u^2 v\right)\right) - \frac{\partial}{\partial x}\left(\tau_x f u\right)$$

$$v = k_u \cdot \rho \frac{\partial u}{\partial y} - k_d \cdot u \frac{\partial \rho}{\partial y} + k_t \cdot \frac{u^2}{(x_t - x)^2} (y_t - y)$$

Here $\rho(x, y, t)$ – the car density, $u(x, y, t)$ – the velocity along the road, $v(x, y, t)$ – the velocity across the road, $P = \alpha \rho^\beta / \beta$ – analogue of the pressure, τ_x – minimum time scale at movement along the road (the time of crossing the given point by several vehicles), τ_y – minimum time scale at movement across the road, $f = a\rho$ – the accelerating/decelerating force, $a = (u_{eq} - u)/T$ – the acceleration, $u_{eq} = u_f \cdot (1 - \rho/\rho_{jam})$ – the equilibrium velocity, $T = t_0 \cdot (1 + r\rho/(\rho_{jam} - r\rho))$ – the relaxation time, $(x_t(x, y), y_t(x, y))$ – the planned destination location, u_f – the free flow velocity, ρ_{jam} – the jam density; α, β, t_0, r, k_u, k_d, k_t – other parameters.

In contrast to previous traffic flow models the variable lateral velocity is introduced in the present research. Traffic flow equations differ from gas dynamic equations by the presence of terms describing the human will. The following strategy of drivers is realized: drivers try to move with the velocity providing the safe traffic; they try to move to the lane with a lower density or a higher velocity; drivers try to reach the planned destination, for example, the needed highway exit. Two last intentions are reflected due to the lateral velocity equation.

One of the modeling results is shown in Figure 1. The road consisting of two lanes has a local widening. At the widening the car density drops down and the velocity increases. But the velocity drops significantly when traffic condenses from three lanes to two lanes. Thus, the traffic capacity of the road decreases.

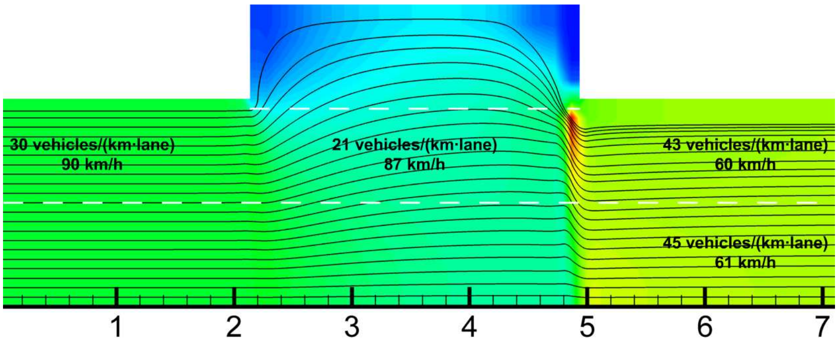

Figure 1: Results of the road widening simulation (the car density is shown by colors)

The authors plan to extend the model to describe road interactions and to simulate multiphase traffic flows when a phase is a group of vehicles characterized by the identical final aim.

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