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Image-Based Computational Homogenization of Random Materials using Level Sets and X-FEM

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ABSTRACT

With the emergence of novel ways of characterizing material microstructures, there is a need to develop computational methods which have the capability to process these material data and solve models with a highly accurate representation of material morphology.

Many approaches have been proposed for the computation of highly heterogeneous materials, which can be classified in two groups. In the first one, statistics are computed from the image for the reconstruction of a material microstructure model. Other methods make a direct use of a digital image, and the finite element method is applied most of the time to solve the boundary value problem. Many authors employ a voxel based mesh, which can be built automatically, but leads to a very large number of elements and to jagged edges. Another way to proceed is to capture geometrical information from the image and then generate a mesh. But this approach can be tedious for 3D complex microstructures.

This paper presents the advantages of using a numerical method that couples the level set method with the extended finite element method (X-FEM), applied to solving problems of homogenization from a digital image representation of the microstructure. One of the key features of this method is the separation between the geometrical data, which are processed by the level set method and the mesh, which does not need to conform the microstructure geometry, thanks to the X-FEM method.

The use of level sets techniques in the context of image segmentation leads to a precise representation of the geometry, the advantage of this technique being that changes in topology are handled implicitly. A user initialized contour is evolved based on image features such as grayscale intensity, gradient magnitude or image edges. The present application uses the Insight Toolkit (ITK) [1] for building the image segmentation application tailored to the specific needs of treating images of material microstructures. For each image the user has to specify the threshold of intensity values and an initial level set model. The level set solution is calculated to subpixel precision, and interpolated at pixel level. For a two phase material, the level set value corresponds to the distance to the material interface, with a positive or negative sign differenciating the phase.

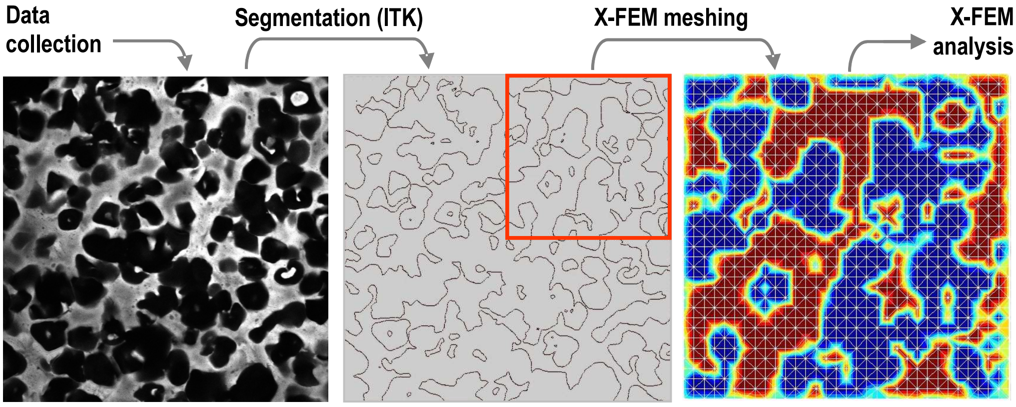

Figure 1: Algorithm: the digital image is segmented using a level set threshold based segmentation filter and the X-FEM mesh is constructed incorporating the level set data in the enriched elements.

The X-FEM method is used to solve the mechanical problem on the microstructure. This formulation does not require a conforming mesh. Rather, a regular or structured mesh is constructed independent of the geometry and the physical boundaries are represented as the iso-zero of the level sets. The elements intersected by the material interface are enriched using a function which represents the strain jump at the interface.

The capabilities of the coupling between the X-FEM and the level set method for micromechanics applications have been shown in [2], but the applications were restricted to microstructures with material interfaces that can be described by analytical level sets. This work advances the method one step further, presenting the process of obtaining the level set information corresponding to a digital image of a material structure and using it in setting up and solving an X-FEM problem, see Fig. 1.

In this work, this numerical framework is used to obtain the overall elastic properties of heterogeneous materials, solving a homogenization problem posed on its microstructure. In order to illustrate the present approach, samples of heterogeneous materials with random microstructure are studied. Since the materials studied here can not be considered as periodic media, their effective properties are computed using an homogenization scheme with homogeneous kinematic boundary conditions. In addition to determining the effective properties of the structure, the question of the size of the representative volume element (RVE) arises. Following the method described in [3], subdivisions of the original sample are considered and the effective properties are determined for each subdivision.

In summary, this study demonstrates the feasibility of using level sets and the X-FEM for treating problems of micromechanics. The method offers a direct way to treat problems of complex microstructures taking advantage of the versatility of level sets and the non-conforming meshing algorithm of X-FEM.

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