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IMPACT PROBLEMS BY TFETI BASED DOMAIN DECOMPOSITION METHOD

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ABSTRACT

The Finite Element Tearing and Interconnecting (FETI) domain decomposition method was introduced by Farhat and Roux [1] as a parallel finite element solver to the self-adjoint elliptic partial differential equations, such as those describing the equilibrium of linear elastic bodies. Its key ingredient is a decomposition of the spatial domain into non-overlapping subdomains that are ‘glued’ by Lagrange multipliers, so that, after eliminating the primal variables, the original problem is reduced to a small, relatively well conditioned, typically equality constrained quadratic programming problem that is solved iteratively. Observing that the equality constraints may be used to define so called ‘natural coarse grid’, Farhat, Mandel and Roux [2] modified the basic FETI algorithm so that they were able to prove its numerical scalability, i.e. asymptotically linear complexity.

The partition of the original domain into subdomains usually generates some ‘floating’ subdomains with not enough prescribed displacements, so that they exhibit rigid body modes and their stiffness matrices are singular. Then implementation of FETI requires the computation of their kernels. However, stable evaluation of the bases of the kernels, though theoretically clear in exact arithmetic, is a bit tricky because of the round-off errors. To overcome this difficulty, Dostál *et al.* [3] proposed to enforce all the Dirichlet boundary conditions by the Lagrange multipliers so that all the subdomains are treated as floating; this version of FETI is referred to as the Total FETI (TFETI). This approach not only removed the problems with identification of the kernels of the subdomain stiffness matrices, but it also resulted in more efficient preconditioning by the ‘natural coarse grid projector’.

Even though FETI and its variants were originally developed for numerical solution to linear elliptic partial differential equations, it was shown that the method is applicable to solution to the contact problems. The algorithms were proposed by Dureisseix and Farhat [4] and Dostál *et al.* [5]. Together with a new class of in a sense optimal algorithms for the solution of quadratic problems with bound and/or equality problems proposed by Dostál and Schöberl [6], this development resulted in a class of

theoretically supported scalable algorithms for contact problems referred to as MPRGP and SMALBE [7].

The TFETI method along with MPRGP and SMALBE algorithms were originally proposed for the solution to steady-state problems. However, removing the prescribed Dirichlet boundary conditions and their replacement by the Lagrange multipliers is also attractive for the solution to dynamic problems. We developed nonlinear dynamic algorithm where we made use of the modules originally intended for solution to the static problems. We show that this new variant of FETI is applicable to solution to impact problems, and that it yields very good results, even with additional geometric and material nonlinearities, while the number of iterations is comparable with the number of iterations for static cases.

We present results of two sets of numerical experiments. The first one is concerned with the impact of two 3D blocks and the second one with the impact of two cylinders with parallel axes. In the former set of experiments we consider the geometric nonlinearities, and in the latter one both the geometric and material nonlinear effects. All numerical experiments were carried out with our in-house general purpose finite element package PMD (Package for Machine Design) [8].

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