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A PARTITION OF UNITY BOUNDARY ELEMENT METHOD FOR FRACTURE AND FATIGUE ANALYSIS

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ABSTRACT

The Boundary Element Method (BEM) has become established as a technique well-suited to the modelling of problems containing cracks. If two opposite and equal tractions are applied on the crack faces then two sets of the same equation are included leading to a singular system. Solutions to this problem include the Method of Subregions where the body is split into two regions with the interface region lying along the crack surface. Then, by enforcing equilibrium of tractions and compatability of displacements a solution can be sought [1]. The Dual Boundary Element Method (DBEM) [2] solves the degenerancy by making use of both the displacement integral equation and the traction integral equation which provide two independent equations for collocation along the crack surface. This removes the need for a multi-domain formulation but places some restrictions on the types of element that can be implemented. Further work has been carried out, though, that allows the method to be used with both continuous and discontinuous elements.

The present work aims to improve the accuracy of the BEM when modelling cracks by implementing the Partition of Unity Method [3]. This allows displacements to be represented in terms of certain enrichment functions multiplied by their associated coefficients. The enrichment functions are chosen from *a priori* knowledge of the local displacement field which in the case of a crack can be derived from the Williams expansion for displacements around a crack tip. These can be written as

$$\{\psi^k(\rho,\theta)\}_{k=1}^{4} \equiv \left\{\sqrt{\rho}\cos\left(\frac{\theta}{2}\right), \sqrt{\rho}\sin\left(\frac{\theta}{2}\right), \sqrt{\rho}\sin\left(\frac{\theta}{2}\right)\sin(\theta), \sqrt{\rho}\cos\left(\frac{\theta}{2}\right)\sin(\theta)\right\} \quad (1)$$

where ρ denotes the distance from the crack tip and θ denotes the angle from the crack plane. The method is very similar to that implemented by Möes et. al.[4] in the Extended Finite Element Method (XFEM) with some key differences in its implementation in the BEM. By representing the displacements in this way improvements in accuracy have been found for computed displacements around the

crack tip and hence similar improvements in accuracy for Stress Intensity Factors (SIFs) and fatigue calculations.

A simple 2D edge-crack example is used to illustrate the method. In this example the enrichment is confined to elements in the vicinity of the crack tip. The displacement for all other elements is expressed in the usual piecewise polynomial basis. Care is taken at the points where unenriched and enriched elements meet. Further work needs to be carried out to determine the choice of elements to be enriched which give the optimum accuracy in results.

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