

## ASYMPTOTIC BEHAVIOR OF A THIN INCLUSION IN AN ELASTIC BODY: THE CASE OF COMPARABLE RIGIDITY

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### ABSTRACT

The communication reports on a research still in progress which analyzes the asymptotic behavior of a two dimensional composite body constituted of two adherent parts separated by a thin adhesive layer of thickness  $\varepsilon$ . The adhesive and the adherents are assumed to behave linearly elastic. Assuming that the elasticity of the adhesive is independent of  $\varepsilon$ , we study the expansion of the energy functional as  $\varepsilon$  goes to zero. The analysis relies upon  $\Gamma$ -convergence techniques. The notion of  $\Gamma$ -convergence has been successfully used in recent years to obtain the limit behavior of thin structures [2, 3, 4, 5] and to derive the interface laws for the adhesive bonding of elastic bodies when the thickness and the elastic coefficients of the glue go to zero [6]. We compare our results with those obtained in [1], where, using matched asymptotic expansions, it is shown that at the first order the glue behaves like a line segment across which displacements and stresses undergo jump discontinuities. The relations between these discontinuities give the interface law of the segment which replaces the thin layer in the limit problem.

### REFERENCES

- [1] R. Abdelmoula, M. Coutris and J. Marigo, “Comportement asymptotique d'une interphase élastique mince”, C. R. Acad. Sci. Paris, Série II *b*, t. **326**, pp. 237-242, (1998).
- [2] E. Acerbi, G. Buttazzo, and D. Percivale, “A variational definition for the strain energy of an elastic string”, J. Elasticity, Vol. **25**, pp. 137-148, (1991).
- [3] G. Anzellotti, S. Baldo and D. Percivale, “Dimension reduction in variational problems, asymptotic developments in  $\Gamma$ -convergence, and thin structures in elasticity”, Asymptotic Anal., Vol. **9**, pp. 61-100, (1994).
- [4] K. Bhattacharya and R.D. James, “A theory of thin films of martensitic materials with applications to microactuators”, J. Mech. Phys. Solids, Vol. **47**, pp. 531-576, (1999).

- [5] H. Le Dret and A. Raoult, "The nonlinear membrane model as variational limit of nonlinear three-dimensional elasticity", *J. Math. Pures Appl.*, Vol. **74**, 549-578, (1995).
- [6] C. Licht and G. Michaille, "A modelling of elastic adhesive bonded joints", *Advances in Mathematical Sciences and Applications*, Vol. **7**, pp. 711-740, (1997).
- [7] F. Lebon and R. Rizzoni, "Asymptotic study of soft thin layer: the non convex case", *Mechanics of Advanced Materials and Structures*, Vol. **15**, pp. 12-20, (2008).
- [8] F. Lebon and S. Ronel, "First order numerical analysis of linear thin layers", *ASME Journal of Applied Mechanics*, Vol. **74**, pp. 824-828, (2007).