

## A NEW STRONG DISCONTINUITY MODELING ELEMENT WITHOUT ADDITIONAL DEGREES OF FREEDOM FOR DELAMINATION PROPAGATION SIMULATIONS

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### ABSTRACT

In this study, the new element named Assumed Displacement Discontinuity-FEM (ADD-FEM) was developed. The element is formulated in order to overcome some drawbacks of existing discontinuous elements, i.e., X-FEM and E-FEM. The ADD-FEM was formulated for the isotropic bilinear 2D element. The error convergence in calculated strain energy release rate obtained by ADD-FEM was compared with that by FEM and E-FEM.

In the past decade, the development of discontinuous elements has made great progress. The eXtended FEM (X-FEM) element has attained popularity among the discontinuous elements. X-FEM has been applied to simulate delamination propagation of carbon fibre reinforced polymer (CFRP) under fatigue loading [1]. The additional degrees of freedom (DOFs) are not desired as they may exceed the maximum allocated memory for the non-linear simulation. Embedded FEM (E-FEM) is another discontinuous element. The main issue of E-FEM is the stress locking behaviour [2].

To overcome the drawbacks of existing discontinuous elements, ADD-FEM is developed. The ADD-FEM was formulated for the isoparametric bilinear 2D element in this study. In the formulation, a through crack within an element was considered. The element with a crack is decomposed into two sub-elements. The sub-element has two master nodes and two dummy nodes. Each dummy node' displacement is expressed as a function of master nodes' displacements by using the assumption of rigid-body rotation. Then, the stiffness matrix of sub-element is condensed to 8x4, i.e., eight equations and four DOFs. By using element equilibrium property of FEM, the number of equations is condensed from eight to four. For each sub-element, 4x4 condensed stiffness matrix is obtained. Upon the reconstruction of stiffness matrix of the element, the translation motion of sub-elements is taken into account by setting the relevant components of the stiffness matrix to zero. For the element which has a crack tip at the boundary, the formulation is slightly changed not to allow crack tip opening displacement.

The error convergence in calculated strain energy release rate (SERR) obtained

by the ADD-FEM was compared with those of standard FEM and EFEM for verification. The strain energy release rate is evaluated by the potential strain energy derivative method since this method is the only one that can be applied to models using all the considered elements. The displacements obtained by analytical solution for pure mode I and mode II were applied to its outer boundary for pure mode I and mode II test, respectively. Figure 1 a) shows the convergence of error in SERR under mode I loading. The ADD-FEM shows very similar convergence behaviour as FEM and all the other elements. Figure 1 b) shows the convergence of error in SERR under mode II loading. The ADD-FEM gives the similar convergence behaviour as FEM again. X-FEM is also compared with those results.

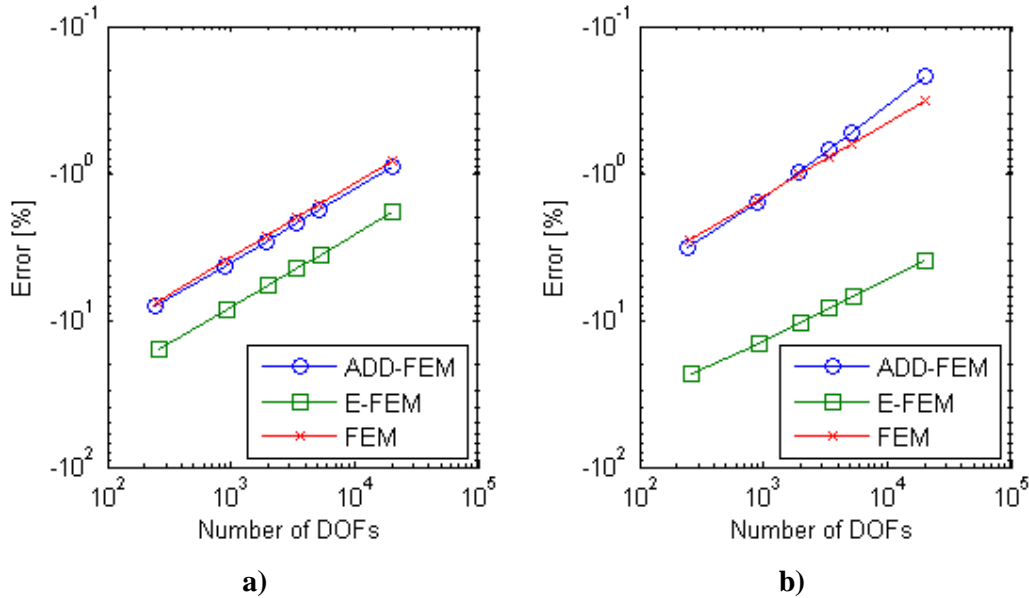


Figure 1. Comparison of error convergence in calculated strain energy release rate:  
a) under pure mode I loading, b) under pure mode II loading.

The error convergence in SERR of the ADD-FEM under mode I and mode II gives very similar behaviour compared with standard FEM. Also, delamination propagation of double cantilever beam specimen made of unidirectional carbon fibre reinforced polymer was simulated by using the ADD-FEM with 2D plane strain assumption. The reaction force-opening displacement relationship is in close agreement with that of experiment.

## REFERENCES

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