

## A new unstructured mesh FE method for the analysis of ship hydrodynamics

\*Julio García-Espinosa<sup>1,2</sup>, Eugenio Oñate<sup>2</sup>

<sup>1</sup> COMPASS Ingeniería y Sistemas, S.A.  
 Tuset 8, 7-2. 08006 Barcelona, Spain  
 Email: info@compassis.com  
 Web page: www.compassis.com

<sup>2</sup> International Center for Numerical Methods  
 in Engineering (CIMNE)  
 Universidad Politécnica de Cataluña  
 Gran Capitán s/n, 08034 Barcelona, Spain  
 E-mail: onate@cimne.upc.edu  
 Web page: www.cimne.upc.es

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### ABSTRACT

The prediction of the performance of ships in still water and waves is a topic of big relevance in the naval engineering. Despite recent advances in computational fluid dynamics, the development of an efficient, accurate and robust numerical algorithm for analysis of this type of problems is still a challenging issue.

This work shows a new stabilized Finite Element Method (FEM) for solving the Navier-Stokes equations including free surface effects that overcomes most of the difficulties of the existing methodologies. The starting point is the modified governing differential equations for an incompressible viscous flow and the free surface condition, incorporating stabilization terms via a Finite Calculus (FIC) procedure [1-4].

To overcome the drawbacks related to the discontinuity of fluid properties across the interface and to impose the interfacial boundary condition we split the domain  $\Omega$  into two overlapping subdomains  $\Omega_1$  (composed by those elements in the air and containing the interface between air and water) and  $\Omega_2$  (composed by those elements in the water and containing the interface between air and water). Based on this subdivision of the domain, we apply a Dirichlet-Neumann overlapping domain decomposition technique with appropriate boundary conditions in order to satisfying the interfacial condition. The corresponding variational problem considering the domain decomposition above described and using a standard notation is:

$$\begin{aligned}
 \Omega_1 \quad & (\rho_1 \partial_t \mathbf{u}_1, \mathbf{v})_{\tilde{\Omega}_1} + \langle \rho_1 (\mathbf{u}_1 \cdot \nabla) \mathbf{u}_1, \mathbf{v} \rangle_{\tilde{\Omega}_1} + (\boldsymbol{\sigma}_1, \nabla \mathbf{v})_{\tilde{\Omega}_1} + \frac{1}{2} (\mathbf{r}_{1m}, (\mathbf{h}_{1m} \cdot \nabla) \mathbf{v})_{\tilde{\Omega}_1} + \\
 & + (\bar{\mathbf{t}}, \mathbf{v})_{\Gamma_{1N}} + (\bar{t}_1 \mathbf{g} + \bar{t}_2 \mathbf{s}, \mathbf{v})_{\Gamma_{1M}} = \langle \rho_1 \mathbf{f}, \mathbf{v} \rangle_{\tilde{\Omega}_1} \\
 & (q, \nabla \cdot \mathbf{u}_1)_{\tilde{\Omega}_1} + \frac{1}{2} (r_{1d}, (\mathbf{h}_{1d} \cdot \nabla) q)_{\tilde{\Omega}_1} = 0 \\
 & \mathbf{u}_1 = \mathbf{u}_2 \quad \text{on } \tilde{\Gamma}_1
 \end{aligned} \tag{1}$$

$$\begin{aligned}
& (\rho_2 \partial_t \mathbf{u}_2, \mathbf{v})_{\tilde{\Omega}_2} + \langle \rho_2 (\mathbf{u}_2 \cdot \nabla) \mathbf{u}_2, \mathbf{v} \rangle_{\tilde{\Omega}_2} + (\boldsymbol{\sigma}_2, \nabla \mathbf{v})_{\tilde{\Omega}_2} + \frac{1}{2} (\mathbf{r}_{2m}, (\mathbf{h}_{2m} \cdot \nabla) \mathbf{v})_{\tilde{\Omega}_2} + \\
\Omega_2 \quad & + (\bar{\mathbf{t}}, \mathbf{v})_{\Gamma_{2N}} + (\bar{t}_1 \mathbf{g} + \bar{t}_2 \mathbf{s}, \mathbf{v})_{\Gamma_{2M}} + (\tilde{\mathbf{t}}, \mathbf{v})_{\tilde{\Gamma}_2} = \langle \rho_2 \mathbf{f}, \mathbf{v} \rangle_{\tilde{\Omega}_2} \quad (2) \\
& (q, \nabla \cdot \mathbf{u}_2)_{\tilde{\Omega}_2} + \frac{1}{2} (r_{2d}, (\mathbf{h}_{2d} \cdot \nabla) q)_{\tilde{\Omega}_2} = 0
\end{aligned}$$

An iterative strategy between the two domains is used to reach a converged global solution in the whole domain  $\Omega$ . It is expected that this global solution will satisfy both restrictions presented above. For this reason we define the following stop criteria:

$$\begin{aligned}
& \|\mathbf{u}_1 - \mathbf{u}_2\| \leq tol_u \quad \text{on } \Omega \\
& |\mathbf{n} \cdot (\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_2) \cdot \mathbf{n} - \gamma \kappa| \leq tol_\sigma \quad \text{on } \Gamma \quad (3)
\end{aligned}$$

The global velocity solution obtained is used as the convective velocity for the transport of the level set function (free surface capturing), used to identify the position of the interface [5,6]. This methodology can be viewed as a combination of domain decomposition and level set techniques. This justifies the name given to the new method: ODDL (by Overlapping Domain Decomposition Level Set).

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