8th. World Congress on Computational Mechanics (WCCM8)
5th. European Congress on Computational Methods in Applied Sciences and Engineering (ECCOMAS 2008)
June 30 – July 5, 2008
Venice, Italy

Leonid A. Rozin [1], Alexander D. Lovtsov[2], *Mikhail S. Smirnov[3]

ELASTICITY PROBLEMS WITH COULOMB FRICTION SOLUTION USING RELAXATION ALGORITHMS

[1]Dr.Sc., Professor, St.Petersburg State Polytechnical University, Russia,194021, St.Petersburg, Bolotnaya St.,16-14
ezlar@LR8192.spb.edu

[2]Dr.Sc., Professor, Pacific State Uiversity, Russia, 680035, Khabarovsk, Bondarya St., 9a-36
lovtsov@bk.ru

[3] Cand.Sc., Senior lecturer, St.Petersburg State Polytechnical University, Russia,197373, St.Petersburg Planernaya St., 47-4-26
smitu@cef.spbstu.ru
http://smitu.cef.spbstu.ru

Key Words: *Coulomb friction, variational statement, relaxation algorithm, numerical analysis, rod on elastic foundation*

ABSTRACT

Problems for elastic rods, interacting with elastic foundations by means of Coulomb friction are considered. Limiting friction forces are supposed given. E.g. the problems of such kind appear in the pipelines on discrete bearings or on continuous foundation analysis.

Well known Coulomb friction conditions [1] are set on the contact between the rod and the foundation. In the simplest case of the straight rod on discrete rigid bearings longitudinal deformation these conditions at bearing *i* of *m* are following:

$$|F_{ti}| \le f_i \cdot |F_{ni}|,\ F_{ni} \le 0,\ \left(|F_{ti}| + f_i \cdot F_{ni}\right)\cdot u_i = 0,\ F_{ti} \cdot u_i \le 0, \qquad (1)$$

where F_{ti}, F_{ni} are frictional and normal forces correspondingly, f_i-friction coefficient, u_i-rod longitudinal displacements.

Using (1) and Lions theorem, the problem under consideration can be formulated in the following variational form:

$$J(u_1, u_2 ,.., u_m) = \inf_{v_1,..,v_m}\left[J_0(v_1, v_2 .. v_m) + \sum_{k=1}^{m} f_k \cdot |F_{nk}| \cdot |v_k| \right], \qquad (2)$$

where J_0 is convex and differentiable Lagrangian for the rod. Functional *J* is also convex, but nondifferentiable because of the caused by friction member, containing the sum.

It is known [1], that problem with contact conditions (1) and, therefore, problem (2) solution exists and is unique on condition that limiting friction forces are given.

Pointwise relaxation algorithm is applied to solve problem (2). Beginning with some $u_i^0, (i=1,..,m)$ and proposing $u_i^n, (i=1,..,m)$ given, the values of $u_i^{n+1}, (i=1,..,m)$ can be consequently determined from the inequality solution:

$$J\left(u_1^{n+1}, \ .., u_{i-1}^{n+1}, u_i^{n+1}, u_{i+1}^{n}, .., u_{i-1}^{n}\right) \leq J\left(u_1^{n+1}, \ .., u_{i-1}^{n+1}, v_i, u_{i+1}^{n}, .., u_{i-1}^{n}\right), \forall\, v_i, i=1,2,.., m. \qquad (3)$$

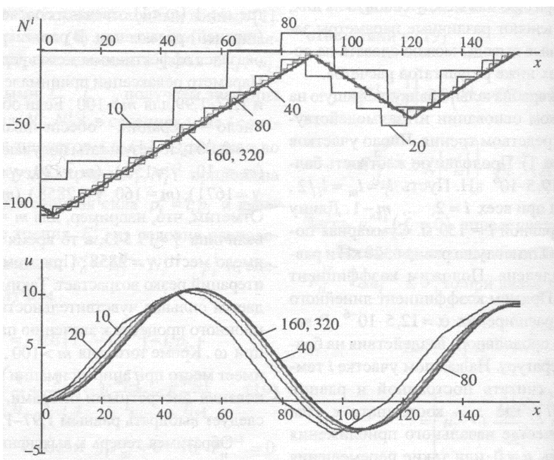
It is proved [2], that pointwise relaxation method converges to problems of type (2) solutions. The effective realization of this method was achieved due to inequality (3) algorithmically exact solution [3].

On each step inequality (3) solution is being found on the base of double-flight rod with boundary bearings *i-1* and *i+1* analysis. Given displacements u_{i-1}^{n+1} and u_{i+1}^{n} let find displacement u_i^{n+1}, considering Coulomb friction on the intermediate bearing *i*.

The numerical estimation of the method rate of convergence, using variants with overrelaxation and under-relaxation, optimal relaxation parameter selection were executed.

Some problems of the same kind were stated and solved. Among them are: rod on discrete compliant bearings longitudinal deformation analysis, rod on discrete rigid and compliant bearings bending analysis, rod on continuous elastic foundation longitudinal and bending deformation analysis and also rods on nonlinear elastic foundations [4] analysis.

In the case of continuous foundation its additional discretization becomes necessary.



Longitudinal forces *N* (upper) and displacements *u* (lower) plots for the problem of rod on continuous rigid foundation with Coulomb friction tension under temperature impact, changing sinusoidally along the rod are shown. Boundary conditions are: $u\big|_{x=0} = 0$, $N\big|_{x=L} = 0$, where *L=159m* is the rod length. The drawing illustrates the algorithm internal convergence (number of bearings- 10, 20, 40, … , 320).

In the problems with nonlinear foundation piecewise-linear approximation of physical characteristics was used. It gave an opportunity to create the algorithm, which can be easily combined with the principal one for solving the problems of type (3).

Conclusions: Variational statements of rods interaction with foundations of different kinds considering Coulomb friction are formulated. Effective numerical solution algorithms for problems of such kind are developed on the base of the common pointwise relaxation scheme. Examples and results discussion are given.

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