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Theories of Scale-Coupling Mechanics and Application of Multiscale Stochastic FEM

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ABSTRACT

Asymptotic theories of classical micromechanics are built on a fundamental assumption of infinite separation of scales. The scale-decoupling assumption however is inapplicable in many circumstances from conventional failure problems to novel small-scale engineering systems. Development of new theories for scale-coupling mechanics is considered to have significant impacts on diverse disciplines. Scale-coupling effects become crucial when size of boundary value problems (BVPs) is comparable to the characteristic length of heterogeneity. Stochasticity, vanishing in deterministic homogenization, resurfaces amid multiscale interactions. Multiscale stochastic modeling is expected to play an increasingly important role in simulation and prediction of material failure and novel multiscale systems such as MEMS and NEMS. In computational mechanics a prevalent issue is, while a fine mesh is desired to achieve high accuracy, a certain mesh size threshold exists below which deterministic finite elements (FE) become questionable. To tackle scale-coupling problems involving stochasticity and the "curse of dimensionality", novel multiscale methods and algorithms are required. Based on a stochastic variational formulation, a multiscale stochastic FE method (MsSFEM) was recently proposed by Xu [1] to tackle multiscale elliptic problems. It has been recognized that, to bring multiscale methods such as the MsSFEM closer to realistic engineering systems, new theories of scale-coupling mechanics are in need.

This work [2] starts investigation of the scale-coupling problems by first looking at uncertainty of material responses due to randomness or incomplete information of microstructures. Classical variational principles are generalized from scale-decoupling micromechanics to scale-coupling BVPs of solid mechanics. Upper and lower variational bounds are provided for probabilistic prediction of material responses. The generalized principles are further applied to quantifying Representative Volume Element (RVE) size effect and threshold of scale separation. Numerical characterization suggests that the size threshold, or equivalently the minimum size of deterministic RVE, is approximately between 10~20 times of "correlation diameter" based on a range of accuracy criteria between 1%~0.1%.

One important implication of the above finding is when FE mesh size decreases to the size threshold, which occurs in many engineering problems such as concrete structures, use of deterministic FEM becomes highly questionable. A bottleneck and challenging problem of stochastic FEM, on the other hand, is its lack of physical connection with real microstructures. By applying the generalized variational principles in stochastic FEM via multiscale formulation, the multiscale stochastic FEM is implemented for multiphase composite materials. Finally the extension to nonlinear theory and computation is also discussed.

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