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MATHEMATICAL PROGRAMS WITH EQUILIBRIUM CONSTRAINTS (MPECs) IN ENGINEERING MECHANICS

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ABSTRACT

The application of mathematical programming concepts for encoding the behaviour of discrete structural systems is a well-developed area and owes much to the pioneering work of Giulio Maier and the late John Munro [see e.g. 1-3].

Research over the last three decades has produced a unified framework for the study of such systems. With its collection of well-constructed algorithms, mathematical programming offers a clear and efficient means of obtaining numerical solutions to a wide range of engineering mechanics problems. Moreover, it invests applications with a refined mathematical formalism which can provide insight into known results and can often also lead to new ones.

An important area of mathematical programming that is particularly beneficial to engineering mechanics is that of complementarity theory [4]. Complementarity describes the perpendicularity of two sign-constrained vectors, and is, moreover, a recurrent and key mathematical structure of many problems in engineering mechanics, such as those involving plasticity and contact-like conditions.

Whilst the formulation and solution of state problems as complementarity systems is common nowadays, less well known is the application, in engineering mechanics, of the class of optimization problems involving complementarity constraints. The latter forms the focus of this presentation.

This challenging class of mathematical programming problems is commonly known as mathematical programs with equilibrium constraints (MPECs). An MPEC is a constrained optimization problem in which some essential constraints are defined by a parameter dependent variational inequality or complementarity system. We will focus on the following MPEC, namely one with parametric complementarity constraints:

$$\begin{aligned} \text{minimize} \quad & f(x,u,v) \\ \text{subject to} \quad & c(x,u,v) \ge 0 \\ & 0 \le u \perp v \ge 0, \end{aligned}$$

where f and c are, respectively, a real-valued and a vector valued function of their arguments, and ⊥ is the complementarity operator, which requires that either a component $u_i = 0$ or the corresponding component $v_i = 0$. Vector x defines the so-called “upper level” variables, whereas (u,v) defines the “lower level” variables.

The most prominent feature of an MPEC and one that distinguishes it from a standard nonlinear program is the presence of complementarity constraints. These constraints classify the MPEC as a nonlinear disjunctive (or piecewise) program. The MPEC is thus equivalent to finitely many smooth nonlinear programs (assuming f and c are smooth), each called a “piece” of the problem. Consequently, besides the common issues associated with a general nonlinear program, the MPEC is complicated by a “combinatorial curse” — a standard feature of all disjunctive problems. The problem class of MPECs has been given a rigorous theoretical treatment only fairly recently by Luo et al. [5]. Even then, there are still no algorithms guaranteed to solve general MPECs, including those that arise in engineering applications.

Maier anticipated the importance of MPECs in engineering mechanics in the 70s, some 20 years before the monograph of Luo et al. [5] was written. In the concluding chapter of the proceedings of the 1977 NATO conference held at Waterloo [1], he describes some potential applications of MPECs to provide, in his words, “… motivation for future research on optimization techniques under complementariy constraints, in view of a variety of future applications …”.

The present talk will present: an introduction to MPECs; an overview of the key difficulties in solving them; various possible nonlinear programming based approaches for solving them; a computational framework, involving a modeling system, for developing and testing algorithms for processing MPECs; and a summary of some recent engineering applications formulated and solved as MPECs. The latter include problems involving parameter identification, minimum weight design, and nontraditional limit analyses.

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