

THE SMOOTHED EXTENDED FINITE ELEMENT METHOD (SmXFEM)

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ABSTRACT

The extended finite element method (XFEM) has emerged as a valid alternative to remeshing for crack propagation problems [1,2] and is now employed with success for three dimensional crack propagation analysis of complex structures [5,6].

The basic idea of XFEM is to add special functions to describe the crack kinematics within the finite elements, so as to avoid the need for a conforming mesh. To introduce the discontinuity, discontinuous functions are added; to help capture the singularity, near-tip fields from the Westergaard asymptotic expansion are used. See [2] for a recent review of the XFEM literature.

Recently, strain smoothing has appeared in the finite element literature and resulted in the discovery of the smoothed FEM (SFEM) [3,4]. The idea is to write the strain field as a spatial average of the compatible strains and use this “smoothed strain” to obtain the element stiffness matrix. This enables the use of polygonal and very distorted meshes and was shown to yield locking-free results for incompressible 2D and 3D elasticity, elasto-plasticity, plate and shells.

In this paper, we combine strain smoothing to the XFEM to obtain the smoothed XFEM (SmXFEM), which shares properties with both the SFEM and XFEM. The integration of the XFEM weak form is performed on the boundary of the split elements, which simplifies implementation, allows dealing with distorted meshes, and arbitrary polygonal meshes. We study the convergence properties of the SmXFEM in the energy norm and analyse its behaviour in incompressible settings.

Additionally, we propose smoothed interaction integrals based on the same idea of strain smoothing and compare the accuracy of the stress intensity factors to those yielded by the standard domain integral method, both with XFEM and SmXFEM discretizations.

For the polygonal version of the XFEM, we propose a new integration technique which suppresses the need for triangulation of the polygons and compare its accuracy with already published methods [7] as well as the polygonal smoothed XFEM.

We show results in 2D linear elastic fracture mechanics where SmXFEM and XFEM are compared, indicating the potential of the proposed approach.

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