8th. World Congress on Computational Mechanics (WCCM8)
5th European Congress on Computational Methods in Applied Sciences and Engineering (ECCOMAS 2008)
June 30 –July 5, 2008
Venice, Italy

Influence of the time step on the stability of the coupling iterations in a partitioned fluid-structure interaction simulation

*** Joris Degroote[1], Peter Bruggeman[2], Robby Haelterman[3], Karen Willcox[4] and Jan Vierendeels[5]**

[1] Ghent University
St.-Pietersnieuwstraat 41
B-9000 Ghent, Belgium
joris.degroote@ugent.be
www.floheacom.ugent.be

[2] Ghent University
Jozef Plateaustraat 22
B-9000 Ghent, Belgium
peter.bruggeman@ugent.be
www.appliedphysics.ugent.be

[3] Royal Military Academy
Renaissancelaan 30
B-1000 Brussels, Belgium
robby.haelterman@rma.ac.be
www.rma.ac.be

[4] Massachusetts Institute of Technology
77 Massachusetts Ave
Cambridge, MA 02139, USA
kwillcox@mit.edu
acdl.mit.edu

[5] Ghent University
St.-Pietersnieuwstraat 41
B-9000 Ghent, Belgium
jan.vierendeels@ugent.be
www.floheacom.ugent.be

Key Words: *Partitioned Fluid-Structure Interaction, Stability Analysis, Time Step.*

ABSTRACT

Fluid-structure interaction can be dealt with using a monolithic or a partitioned approach. In the monolithic approach, both the fluid and structure problem are solved with a single code [1], whereas in the partitioned approach the fluid and structure are solved with separate codes. The partitioned approach to fluid-structure interaction and more specifically the effect of the time step on the stability of the coupling iterations in a time step is considered in this paper.

Previous work [2] showed that the number of coupling iterations in a partitioned simulation increases when the time step decreases and no other parameters are changed. This behaviour is studied by means of a one-dimensional model of unsteady flow in a flexible tube. The fluid is incompressible and inviscid and gravity is not taken into account. The velocity is imposed at the inlet and a non-reflecting pressure outlet boundary condition is used. The structure model contains no mass, as the inertia of the tube wall is neglected with regards to that of the fluid. All parameters are identical to those of the two-dimensional model for the pulsating flow in an artery in Vierendeels et al. [2], unless mentioned otherwise.

The tube is divided in N segments of length Δx. The cross sectional area a_i of segment i of the tube is related to the kinematic pressure p_i by

$$a_i = a^* \left(\frac{p^* - 2c_{mk}^2}{p_i - 2c_{mk}^2} \right)^2 \tag{1}$$

with c_{mk} the Moens-Korteweg wave speed which is a measure for the stiffness of the tube. The asterisk indicates the reference state.

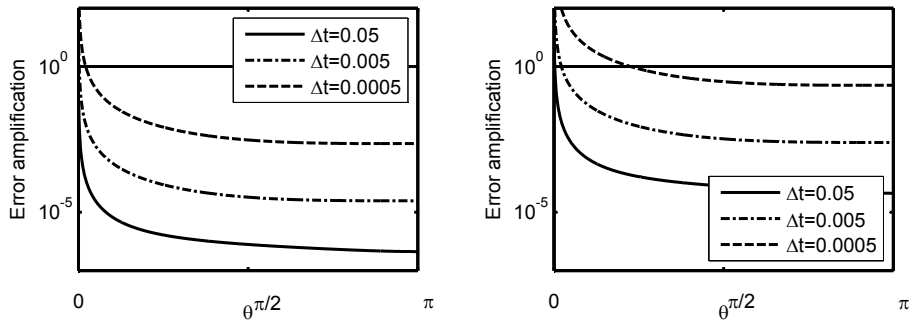

Figure 1: Error amplification as a function of θ for (a) $c_{mk} = 11$ m/s and (b) $c_{mk} = 1.1$ m/s.

The flow equations are the conservation of mass and momentum. Central discretization is used, except for the convective term in the momentum equation which is discretized with a first-order upwind scheme, giving

$$\frac{\Delta x}{\Delta t}\left(a_i - a_i^n\right) + u_{i+\frac{1}{2}} a_{i+\frac{1}{2}} - u_{i-\frac{1}{2}} a_{i-\frac{1}{2}} - \alpha(p_{i+1} - 2p_i + p_{i-1}) = 0 \tag{2a}$$

$$\frac{\Delta x}{\Delta t}\left(u_i a_i - u_i^n a_i^n\right) + u_i u_{i+\frac{1}{2}} a_{i+\frac{1}{2}} - u_{i-1} u_{i-\frac{1}{2}} a_{i-\frac{1}{2}} + a_{i+\frac{1}{2}} \frac{p_{i+1} - p_i}{2} + a_{i-\frac{1}{2}} \frac{p_i - p_{i-1}}{2} = 0 \tag{2b}$$

for $u_i \geq 0$. u is the velocity along the axis of the tube, Δt the time step and $\alpha = a^* / \left(u^* + \Delta x / \Delta t\right)$ the coefficient of the pressure stabilization term. The subscript i indicates the cell centres and all variables are on time level $n + 1$, except for those indicated with the superscript n.

The stability of fixed point iterations between the flow and the structure is studied by linearizing the equations (1) and (2) and performing Fourier error analysis, with ω the spatial frequency of the error modes. The error amplification as a function of $\theta = \omega \Delta x$ is shown in Figure 1 for different values of Δt and c_{mk} and it can be seen that the error amplification increases as Δt decreases.

For the configuration with $c_{mk} = 11$ m/s in Figure 1(a), the error amplification is smaller than 1 except at $\theta = 0$ for $\Delta t = 0.05$ and 0.005 so fixed point iterations should be stable, which is confirmed by non-linear simulations which required respectively 3.68 and 5.85 coupling iterations on average. The simulation with $\Delta t = 0.0005$ failed because the error amplification is bigger than 1 for $\theta > 0$. For the same reason, the cases with $c_{mk} = 1.1$ m/s in Figure 1(b) cannot be simulated with fixed point iterations. If coupling with reduced-order models [2] is used, the simulations with $c_{mk} = 11$ m/s all required around 4 coupling iterations as only few modes become unstable due to the decreasing time step. The simulations with $c_{mk} = 1.1$ m/s required on average 4.96, 7.58 and 15.39 coupling iterations for $\Delta t = 0.05$, 0.005 and 0.0005 s, as the error amplification becomes bigger than 1 for more modes if Δt decreases.

Conclusion If the structure in a partitioned FSI simulation is so flexible that error modes become unstable as Δt decreases, then more coupling iterations are required as Δt decreases.

J. Degroote and P. Bruggeman gratefully acknowledge a Ph.D. fellowship of the Research Foundation - Flanders (FWO). J. Degroote gratefully acknowledges a grant for a long stay abroad at the Massachusetts Institute of Technology of the FWO.

REFERENCES

[1] M. Heil. "An efficient solver for the fully coupled solution of large-displacement fluid-structure interaction problems". *Comput. Meth. Appl. Mech. Eng.*, Vol. **193**, 1–23, 2004.

[2] J. Vierendeels, L. Lanoye, J. Degroote and P. Verdonck. "Implicit coupling of partitioned fluid–structure interaction problems with reduced order models". *Comput. Struct.*, Vol. **85**, 970–976, 2007.