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Hybrid solvers for heterogeneous multiscale methods

A. Abdulle[1]

[1]School of Mathematics,
University of Edinburgh
JCMB, King's Buildings
Edinburgh EH9 3JZ, UK
a.abdulle@ed.ac.uk,
http://www.maths.ed.ac.uk/people/show/person/80

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ABSTRACT

The past few years have seen increasingly growing research activities aiming at developing novel multiscale computational methods. While traditional approaches were based on sequential strategies with empirical macroscopic models derived with parameters computed beforehand from microscopic models, new methods based on simultaneous coupling or ”on the fly” computations, extracting coarse dynamics from multiscale systems have emerged.

In this talk we discuss hierarchical finite element methods (FEMs) based on a coupling of macro and micro solvers constructed in the framework of the heterogeneous multiscale method (HMM). This is a new mathematical framework recently introduced by W. E and B. Engquist for constructing and analyzing multiscale algorithms.

Many fundamental numerical and modeling issues arise, when coupling a hierarchy of solvers. One such issue is the propagation of errors of a micro model to the macroscale. To address this question, a fully discrete analysis of the heterogeneous multiscale FEM has been proposed in [1]. Another issue is related to the type of micro and macro solvers used in a hierarchical approach as the HMM. In [2], a hybrid method combining a FEM for the macro solver with a pseudo-spectral method for the micro solver has been proposed. This method achieves near optimal computational complexity for problems involving oscillatory coefficients with sufficient regularity.

For some problems, local conservation properties in the numerical approximation and flexibility in meshing (e.g. hanging nodes, local refinement) are desirable. To obtain such properties, numerical methods based on (local) approximations as the discontinuous Galerkin (DG) FEM have become increasingly popular. Such methods have been extensively studied for hyperbolic problems, advection-diffusion and diffusion problems. In [3], a hybrid multiscale method using a DG-FEM at the macro level has been proposed and analyzed, allowing the aforementioned local conservation properties and flexibility in meshing for the macroscopic dynamics.

We will review these recent progress in the numerical analysis of macro-to-micro finite element methods based on the heterogeneous multiscale method (HMM). Examples and applications will be considered.

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