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TRANSIENT HEAT DIFFUSION ANALYSIS USING A COLLOCATION METHOD WITH MODIFIED EQUILIBRIUM ON LINE METHOD FOR IMPOSITION OF NEUMANN AND ROBIN BOUNDARY CONDITIONS

A. Sadeghirad [1], I. Mahmoudzadeh Kani [2] and A. Vaziri Astaneh [3]

[1] Ph.D. Student
School of Civil Engineering,
University of Tehran,
P.O.Box 11365-4563, Tehran, Iran
arsadegh@ut.ac.ir
ar_srad@yahoo.com

[2] Associate Professor
School of Civil Engineering,
University of Tehran,
P.O.Box 11365-4563, Tehran, Iran
imkani@ut.ac.ir
eng.ut.ac.ir/Fa/CIV/imkani

[3] B.Sc. Student
School of Civil Engineering,
University of Tehran,
P.O.Box 11365-4563, Tehran, Iran
vaziri@gmail.com

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ABSTRACT

The attractive advantage of the collocation-based meshless methods is that they are simple to implement and have less computational cost in comparison to the weak form-based methods. However, collocation-based meshless methods are often found less accurate and most importantly they are usually suffering from instability, especially for problems governed by partial differential equations with derivative boundary conditions For alleviating this defficiency and improving the solution accuracy of the collocation-based meshless methods several techniques have been proposed so far [such as 1-4].
Recently, Sadeghirad and Mohammadi [5] proposed the equilibrium on line method (ELM) for imposition of Neumann boundary conditions in the FPM. The ELM presents a formulation for satisfying the Neumann boundary conditions by a set of weak form equations consistent with the governing equations. In the ELM, for any Neumann node, test domain is the line that connects two mid points of distances between the node in question and the two neighboring nodes on Neumann boundary. In order to obtain the weak form equations, the governing differential equations in the local coordinate system are integrated over these lines. For any of these lines, a balanced area (B-Area) constructed by dragging the line along the unit inward normal vector is defined [5]. Furthermore, the Heaviside function is used as a weight in these integrals. This method is a 'truly' meshless one as it needs no mesh, neither for the purpose of interpolation of the solution variables, nor for the integration of the equilibrium on lines.
In this work, a modified version of the ELM is presented. In this method, a modified procedure for choosing the integration domains is proposed. Based on this procedure, equilibrium on two lines that connect the node in question to the two neighboring nodes on Neumann (or Robin) boundary is satisfied in the global coordinate system for any

node located on the Neumann (or Robin) boundary. Main advantage of this procedure is its ability to represent the geometry of the Neumann boundary more accurately in comparison to the original ELM (Figure 1). Moreover, application of different test functions is examined. Application of these weight functions results in an improvement in the modified version of ELM in comparison to the original ELM.

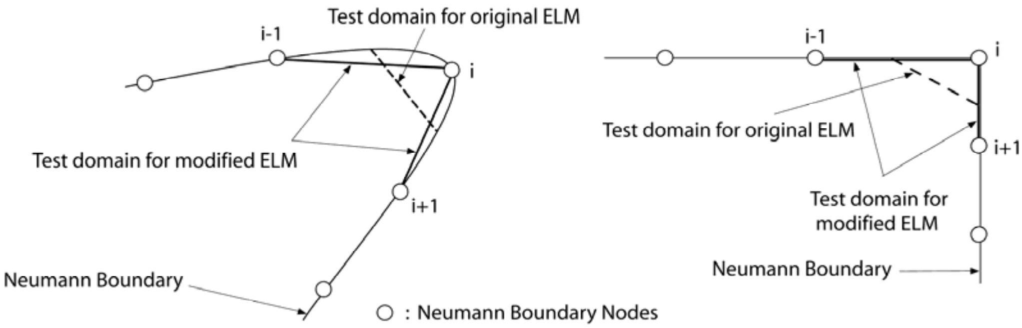

Figure 2. Representation the Neumann boundary at node $\mathbf{x}_i$ located on the Neumann boundary.

The performance of the modified version of the ELM is studied for collocation methods based on two well-known schemes to construct meshless shape functions: moving least squares (MLS) approximation and locally supported radial basis point interpolation method (RPIM). Three numerical examples of two-dimensional unsteady Fourier heat conduction are presented as follows: (1) unsteady heat diffusion in the square domain, (2) unsteady heat diffusion in a quarter circular plate, (3) unsteady heat diffusion example with the Robin type boundary condition.

These examples demonstrate the stability, accuracy and convergence of the proposed method. Based on these examples, the results obtained from the modified ELM are more accurate and stable than those obtained from the direct collocation method and the original ELM. One of the most important parameters in the modified ELM is the ratio of width to length of the B-Areas. Based on the numerical investigations in this paper, it is concluded that the values between 0.1 and 0.5 are appropriate for this ratio. Moreover, it can be concluded that by using the considered test functions (Gaussian, 4th order spline and 3rd order polynomial) an improvement in results is observed in comparison to those obtained using the Heaviside test function.

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