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Selective array imaging of cracks in homogeneous and random media

Liliana Borcea[1], George Papanicolaou[2] and *Chrysoula Tsogka[3]

[1] Computational and Applied Mathematics, MS 134, Rice University
6100 Main Street, Houston, TX 77005-1892, USA
borcea@caam.rice.edu

[2] Department of Mathematics, Stanford University
Stanford CA 94305, USA
papanicolaou@stanford.edu

[3] Applied Mathematics Department, University of Crete & Institute of Applied and Computational Mathematics Foundation for Research and Technology - Hellas
Heraklion 71409, GREECE
tsogka@tem.uoc.gr

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ABSTRACT

We consider here selective array imaging of the edges of scatterers in homogeneous and random media using the singular value decomposition of the response matrix. For homogeneous media, the problem of selective array imaging was considered in [2, 3]. By computing analytically the SVD of the response matrix in the Fraunhofer diffraction regime, it has been shown [2, 3] that information about the edges of the scatterers is contained in the singular vectors corresponding to singular values that are in the transition zone between the large ones and zero. The diffraction regime that we consider in the numerical simulations (see also [3]) corresponds to ultrasound non-destructive testing. Although, this is not in the Fraunhofer diffraction regime, the results that we obtain numerically are qualitatively the same as the ones predicted by the theory. The imaging method that we describe next holds for general extended scatterers. In the numerical simulations we consider its application to cracks.

The imaging method We wish to image a scatterer using an array of N_s sources and N_r receivers. The data is the array impulse response matrix, denoted $\widehat{\Pi}(\vec{\mathbf{x}}_r, \vec{\mathbf{x}}_s, \omega)$ for ω in the frequency band $\omega_0 + [-B/2, B/2]$. Here $\vec{\mathbf{x}}_j$ is the location of the array element j, ω_0 the central frequency and B the bandwidth. To image selectively the edges of the scatterer, the data are filtered as follows ($\mathbf{v}^*$ denotes the complex conjugate transpose of $\mathbf{v}$)

$$D(\omega, d)\widehat{\Pi}(\omega) = \sum_{j=1}^{N} d_j(\omega)\mathcal{P}_j(\omega)\widehat{\Pi}(\omega) = \sum_{j=1}^{N} d_j(\omega)\sigma_j(\omega)\widehat{\mathbf{u}}_j(\omega)\widehat{\mathbf{v}}_j^*(\omega), \tag{1}$$

with $\mathcal{P}_j(\omega)$ being the projection matrices

$$\mathcal{P}_j(\omega) = \widehat{\mathbf{u}}_j(\omega)\widehat{\mathbf{u}}_j^\star(\omega) \tag{2}$$

onto the space spanned by the j-th left singular vector and

$$d_j(\omega) = \begin{cases} 1 & \text{if } j \in J(\omega) \\ 0 & \text{otherwise .} \end{cases} \tag{3}$$

In (1), $\mathbf{u}_j(\omega)$ and $\mathbf{v}_j(\omega)$ are the left and right singular vectors of the response matrix and $\sigma_j(\omega)$ are its singular values. The set $J(\omega)$ determines which singular vectors we keep. To image in homogeneous media we use a selective migration (SM) method,

$$I_{SM}(\vec{\mathbf{y}}^S) = \int d\omega \sum_{s=1}^{N_s} \sum_{r=1}^{N_r} \exp\left[i\omega\tau(\vec{\mathbf{x}}_r, \vec{\mathbf{y}}^S) + i\omega\tau(\vec{\mathbf{x}}_s, \vec{\mathbf{y}}^S)\right] \overline{\left(D(\omega, d)\widehat{\Pi}(\omega)|\right)_{r,s}} \tag{4}$$

with $\vec{\mathbf{y}}^S$ the search point and $\tau(\vec{\mathbf{x}}, \vec{\mathbf{y}})$ the travel time between points $\vec{\mathbf{x}}$ and $\vec{\mathbf{y}}$. In (4) overline denotes complex conjugate.

If we keep all the singular vectors in (4), i.e., $J(\omega) = 1, \ldots, N$, our filter is the identity and (4) becomes the usual Kirchhoff migration (KM) method [1]. If we only keep the singular vector that corresponds to the largest singular value, i.e., $J(\omega) = 1$, we can image the strongest part of the scatterer which is adventegeous for detection but not problematic for imaging as it does not give any information about the shape of the scatterer. This is closely related to the DORT method [5]. To image the edges of the scatterer, the theory suggests [2,3] to keep the singular vectors corresponding to singular values that are between the large ones and zero. Therefore $J(\omega)$ is chosen so that the normalized singular values $\sigma_j(\omega)/\sigma_1(\omega)$ of $\widehat{\Pi}(\omega)$ belong to some interval $[a, b]$ with $1 > a > b > 0$,

$$J(\omega; [a, b]) = \left\{ j \left| \frac{\sigma_j(\omega)}{\sigma_1(\omega)} \in [a, b] \right. \right\}. \tag{5}$$

We also consider imaging in random media, i.e., media with inhomogeneities that are unknown and cannot be estimated in detail. As explained in [2] (see also references therein) in such media there is significant multiple scattering of the waves by the inhomogeneities, and usual migration methods create images with speckles that are difficult to interpret. These images are also unstable, which means that they change unpredictably with different realizations of the random medium. To stabilize the imaging process in random media, we introduced in [4] the coherent interferometric (CINT) imaging method which is a smoothed version of travel-time (Kirchhoff) migration.

We will present in the talk the CINT version of the imaging functional (4) and we will show numerical results for selective imaging of cracks in homogeneous and random media.

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