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# Adaptive discretization of parameter identification problemes in PDEs

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## ABSTRACT

Parameter identification problems for PDEs such as

*Find $q \in Q$ from observations $g^\delta$ of $C(u)$ of the solution $u$ to the PDE (in weak form)*

$$A(q,u)(v) = (f,v) \quad \forall v \in V$$

often lead to large scale inverse problems. To reduce the computational effort for the repeated solution of the forward and even of the inverse problem — as it is required for determining the regulrization parameter according to the discrepancy principle in Tikhonov regularization — we use an adaptive discretization based on goal oriented error estimators. This concept originating from optimal control provides an estimate of the error in a so-called quantity of interest — a functional of the control $q$ (which in our context is the searched for parameter) and the PDE solution $u$ — based on which the discretizations of $q$, $u$ (and possibly also the adjoint PDE solution) are locally refined. The crucial question for parameter identification problems is now the choice of an appropriate quantity of interest.

A convergence analysis on discretized spaces for $q$ and $u$ of the Tikhonov regularization

$$\text{Minimize} \quad J_\alpha(q,u) = \|C(u) - g^\delta\|^2 + \alpha\|q\|^2 \text{ over } q \in Q\,, \quad u \in V\,,$$
$$\text{under the constraints } A(q,u)(v) = (f,v) \quad \forall v \in V$$

with the discrepancy principle

$$\|C(u(q^\delta_{\alpha_*})) - g^\delta\| = \tau\delta$$

for regularization parameter choice shows, that in order to determine the correct regularization parameter, one has to guarantee sufficiently high accuracy in the squared residual norm $\|C(u(q^\delta_{\alpha_*})) - g^\delta\|^2$ which is therefore our quantity $i$ of interest — whereas $q$ and $u$ themselves need not be computed precisely everywhere. This fact allows for relatively low dimensional adaptive meshes and hence for a considerable reduction of the computational effort. In addition to that, once the error estimate

has been evaluated, the special structure of the Tikhonov functional leads to a simple computation of the derivative $i'$ of the squared residual norm with respect to the regularization parameter $\alpha$, as we require it for carrying out Newton's method for determining $\alpha$ according to the discrepancy principle. Investigating into the accuracy requirements for a fast convergence of this Newton iteration and providing error estimators also for $i'$, we arrive at a highly efficient method for determining the regularization parameter. To finally compute a minimizer of the Tikhonov functional $J_{\alpha_*}$ with the so determined regularization parameter $\alpha_*$ we can again use goal oriented adaptivity, this time with $J_{\alpha_*}$ as a quantity of interest.

## REFERENCES


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