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Justification of the periodic distribution of transverse cracks in a composite by energy minimization

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ABSTRACT

The goal of this paper is to explain the periodic distribution of transverse cracks in layered composites using energetic considerations. For this purpose, we mimimize the total energy of the composite by adopting the point of view of Griffith for the surface energy. This work contains two parts. In the first part we develop a semi-analytical model in which we assume a priori that the cracks are transverse and look for their optimal spatial distribution. The main result is that we find two possible regimes for the crack layout depending on the parameters of the composite. In the second part we compute the optimal response without any a priori hypotheses on the cracks geometry. Specifically, we consider a cylindrical composite domain of length L (along the x-axis) and circular cross-section of area S. Its core, made of an elastic unbreakable core (with cross-sectional area $cS, 0 < c < 1$, Young's modulus E_f, and Poisson's ratio ν_f) of circular-cross section, is surrounded by a brittle elastic annulus (with cross-sectional area $(1 - c)S$, Young's modulus E_m, Poisson's ratio ν_m, and fracture toughness k_m). The annulus is perfectly bonded to the core. The sample is clamped at its far left cross-section and submitted to a monotonically increasing displacement load δ at its far right cross-section.

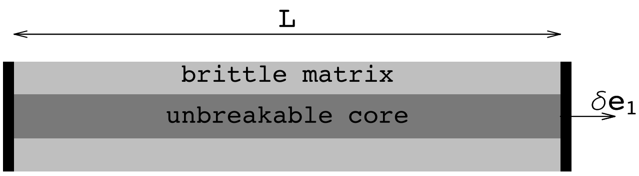

Figure 1: Composite tube in traction

1. Theoretical analysis. We only assume that the only possible crack states are annular cracks through the entire matrix. For a given displacement δ, we minimize the total energy (sum of the elastic energy and the Griffith surface energy) with respect to the number of cracks and their location. The elastic energy involves the effective compliance $\mathcal{S}(l)$ of a cylinder of length l with a centered transverse crack. The spatial distribution of the cracks depends on the concavity of the function $l \mapsto \mathcal{S}(l)$. If this function is concave, which is the case for large values of the ratio E_f/E_m, then the transverse cracks are periodically distributed, their number grows with δ, cf Figure 2 (left top and left bottom). On the other hand, if $\mathcal{S}(l)$ is not concave, which is the case for small values of the ratio E_f/E_m, then the periodic cracks arise only on a part of the cylinder, the other part remaining uncracked. Moreover the dimensionless period $l_{\#}$ is a characteristic length of the composite obtained from the graph of $\mathcal{S}(l)$, cf Figure 2 (right top and right bottom).

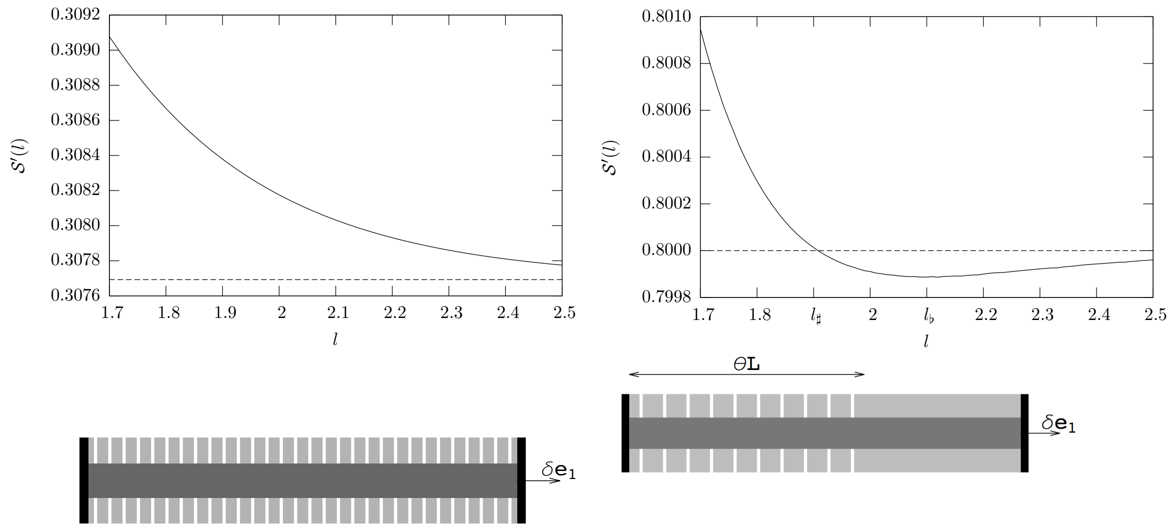

Figure 2: Left: the concavity of $\mathcal{S}$ for $E_f/E_m = 10, \nu = 0.25, c = 0.3$ and the induced periodic cracking. Right: the non-concavity of $\mathcal{S}$ for $E_f/E_m = 2, \nu = 0.25, c = 0.3$ and the second regime of transverse cracking

2. Numerical simulation by the finite element method and elliptic regularization of the Griffith energy. The computations are made using the method presented in [1] and based on the elliptic regularization of the Griffith model. No a priori assumptions are made on the evolution of the cracks in space and time. Depending on the values of the parameters of the composite we can even account for the competition between debonding and transverse cracking, cf Figure 3. Several computations will be presented.

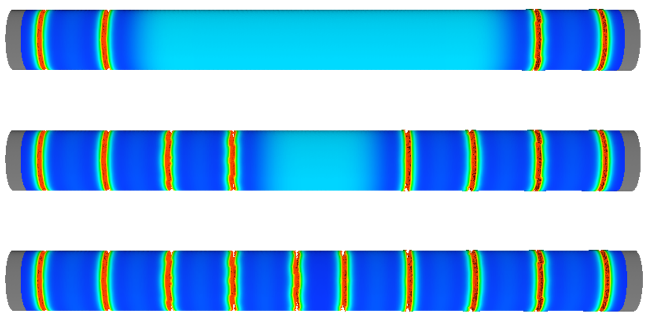

Figure 3: Numerical example for $E_f/E_m = 10$ and three values of δ: δ =31.7, 32.4, 35.4.

REFERENCES

[1] B. Bourdin, G.A. Francfort and J.-J. Marigo. The variational approach to fracture. Springer (2007).