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FREE BOUNDARY CONDITIONS FOR HIGH-ACCURACY AEROACOUSTIC ALGORITHMS

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ABSTRACT

Numerical simulation of aeroacoustic problems requires discrete approximations to governing equations having high accuracy. On uniform spatial grids, finite differences preserving dispersion relations are widely used, including DRP schemes by Tam [1] and their modifications by Bailly and Bogey [2]. The time advancing is performed mainly with the use of explicit Runge–Kutta methods.

In the algorithms mentioned, all spatial derivatives are approximated on extended stencils involving 7 points in [1] and up to 13 points in [2]. This fact prevents of using homogeneous algorithms for calculations in all internal mesh nodes. A common issue is in applying biased differences or resorting to reduced stencils near free boundaries. In this study, we consider all the equations specified in “non-standard” nodes as boundary conditions of special kind. We deal with a class of *discrete nonreflecting* boundary conditions.

We start with the linear one-dimensional transport equation

$$\partial u/\partial t + \partial u/\partial x = 0. \tag{1}$$

Its spatial discretizations have a form shown here for the DRP scheme,

$$du_j/dt + \frac{1}{h}\sum_{l=-3}^{3} a_l u_{j+l} = 0, \quad j = 3, \ldots, N-3. \tag{2}$$

Such numerical schemes, unlike the differential case (1), describe several types of waves. The worst kind of spurious waves are sawtooth grid oscillations propagating in the opposite direction. The technique to construction of nonreflecting boundary conditions consists of an accurate reproduction of dispersion relations suggested by internal scheme patterns. Near the right-hand boundary, the discrete “physical” mode is approximated on non-symmetric stencils, and near the left-hand one, the spurious sawtooth wave is modeled. For DRP scheme (2), we obtain a set of left-hand boundary conditions

$$du_j/dt + \frac{1}{h}\sum_{l=0}^{6} c_{jl} u_l = 0, \quad j = 0, 1, 2. \tag{3}$$

Such approach distinguishes from the traditional grid approximations in that both initial and target objects have discrete nature.

The resulting boundary equations are not uniquely determined. We calculate the coefficients based on the maximal approximation order as well as on least-square optimizations of dispersion relations. E.g., the values of c_{jl} from (3) can be obtained as the solution to the problem

$$\int_0^L \left| \sum_{l=0}^{6} (-1)^{l-j} c_{jl}\, e^{i(l-j)\varphi} - \sum_{l=-3}^{3} (-1)^l a_l\, e^{il\varphi} \right|^2 d\varphi \;\rightarrow\; \min$$

as $$\sum_{l=0}^{6} (-1)^{l-j} c_{jl} = \sum_{l=-3}^{3} (-1)^l a_l\,, \quad \sum_{l=0}^{6} (-1)^{l-j} (l-j)^n c_{jl} = \sum_{l=-3}^{3} (-1)^l l^n a_l\,, \;\; n = 1, 2, 3, 4.$$

The versions of boundary conditions constructed are tested from the stability viewpoint by means of numerical and theoretical investigations of spectra of finite-difference operators.

The similar technique is applied to the quasilinear transport equation

$$\partial u/\partial t + u\,\partial u/\partial x = 0$$

and the Euler system of equations linearized with respect to a constant background flow. Some new situations are dealt with. Maintaining stability of the schemes is especially important in these cases.

Boundary conditions for the Euler equations are typically reduced to problems for the scalar transport equation. The choice of appropriate boundary conditions should obey the following recommendations.

- Avoid using downwind differences (with respect to characteristics).
- In near-boundary internal nodes, approximate some differential nonreflecting boundary conditions rather than the governing equations.

The algorithms proposed are validated on numerical simulation of test problems.

REFERENCES

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