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ADAPTIVE MODELING OF TURBULENT FLOW BASED ON WEAK EULER SOLUTIONS

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ABSTRACT

Computer simulation of turbulent fluid flow is based on the Navier-Stokes equations, a mathematical model for which no proof of existence and uniqueness of a classical solution is known. For the incompressible Navier-Stokes equations the existence of a weak solution, satisfying the equations in mean value, was proven by Leray in 1934, but no result on uniqueness is available for weak solutions, and the extension to a classical solution remains an open problem. For incompressible flow the only empirical parameter in the model is the viscosity ν, and to characterize the relative importance of viscous and inertial effects in the flow we may define a Reynolds number Re=UL/ν, with U and L typical velocity and length scales, respectively. The Euler equations correspond to the case of zero viscosity (infinite Reynolds number), typically viewed as the limit of the Navier-Stokes equations with vanishing viscosity. The Euler equations lack any empirical parameters, and the mathematical problem of existence and uniqueness is even less clear, where not even the existence of a weak solution to the Euler equations can be proved.

A main problem is now what information about turbulent flow we can compute from the Navier-Stokes/Euler models? An established approach is to seek modified equations describing the average flow field using subgrid/turbulence modeling in LES/RANS equations [1], leading to the well known open problem of closing the equations in terms of the mean field only. Many different subgrid/turbulence models have been proposed but are in general very sensitive for changes in data and/or numerical method, and thus reliability is a problem. Recently, awareness of this problem has increased, and efforts are made towards error control in LES, including the modeling error of the subgrid model [3].

An alternative approach to LES/RANS is suggested in [2], introducing the mathematical concepts of ϵ-weak solutions and weak uniqueness, where no subgrid/turbulence modeling is used but instead approximate weak solutions are computed using a stabilized finite element method, referred to as a General Galerkin G2 method, for which weak output uniqueness in a chosen output of interest can be proven using dual/adjoint techniques. Thus in the G2 approach the uniqueness of a mathematical limit of G2 solutions itself is not of interest, but only the uniqueness of certain functionals of such G2 sequences. A

typical example may be computational approximation of the turbulent flow past a vehicle, for which the output (functional) of interest may be the drag force. An adaptive algorithm is then designed to modify the underlying G2 solution to improve the approximation of drag, typically by mesh refinement.

In this paper we present new theoretical and computational results for the G2 method applied to the Euler equations as a model for very high Reynolds numbers, which we refer to as an Euler/G2 or EG2 method, with focus on complex problems, verification and validation with respect to experimental results, and boundary conditions for turbulent boundary layers. Extensions to compressible flow and fluid-structure interaction are presented in separate papers at this conference.

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