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Fracture criterion for piezoelectric ceramics using the exact boundary conditions applied to the crack surfaces

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ABSTRACT

Piezoelectric ceramics are widely used as sensors and actuators in smart structures, despite the absence of fundamental understanding of their fracture behavior. Piezoceramics are brittle and susceptible to cracking. Because reliability of these devices is important, there has been tremendous interest in studying their fracture and failure behavior. For piezoelectric material, in addition to the usual three modes of fracture, there is a fourth mode associated with the electric field. Thus, K_I, K_{II} and K_{III} are the stress intensity factors and K_{IV} is the electric flux density intensity factor. Therefore, these parameters will be denoted as intensity factors.

The aim of this investigation is to develop a mixed mode fracture criterion for piezoelectric ceramics utilizing the exact crack face boundary conditions. This criterion is based upon the energy release rate and one or two phase angles, determined from the ratio between the intensity factors. To examine this criterion, experimental results of Jelitto et al. [1] were analyzed by means of the finite element method and an M-integral in which the exact boundary conditions are applied to the crack faces [2].

There are four approaches in the literature for describing boundary conditions on the crack faces for piezoelectric materials, i.e. impermeable, permeable, semi-permeable and exact. With the impermeable crack assumption, the permittivity of the gap is taken to be zero. This implies that the normal component of the electric flux density must vanish there. In contrast, with the permeable model, the crack is assumed not to perturb the electric field so that both the electric potential and normal electric flux density are continuous across the crack faces. Semi-permeable boundary conditions were proposed by Hao and Shen [3], in which the electric permeability in the crack gap is accounted for. In each of these three models, it is assumed that the crack faces are traction free. Landis [4] proposed a new set of boundary conditions that consists of additional crack closing tractions; these conditions are called the exact boundary conditions and were used here as part of an M-integral [2].

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An iterative framework was developed for analyzing a more realistic model of a crack with the exact boundary conditions applied to it [2]. Finite element analyses on four-point bend PIC-151 specimens were carried out according to test results obtained by Jelitto et al. [1]. In these experiments, the poling direction was perpendicular to the crack faces and both mechanical loads and electrical fields were applied. The M-integral was employed to calculate intensity factors (for more details see [2]) in each problem. With the iterative procedure, at each step k the normal displacement u_y and electric potential ϕ were obtained on the crack faces. At step $k+1$, the normal electric flux density D_y and stress σ_{yy} were calculated as [2]

$$D^+_{y\ (k+1)} = D^-_{y\ (k+1)} = -\kappa_a \left(\frac{\phi^+ - \phi^-}{u_y^+ - u_y^-} \right)_{(k)} \tag{1}$$

$$\sigma^+_{yy(k+1)} = \sigma^-_{yy(k+1)} = \frac{1}{2}\kappa_a \left(\frac{\phi^+ - \phi^-}{u_y^+ - u_y^-} \right)^2_{(k)} \tag{2}$$

where $^+$ and $^-$ denote the component at the upper and lower crack faces, respectively, and κ_a is the dielectric permittivity inside the crack gap. For the first iteration, impermeable crack boundary conditions were enforced, so that the values of the normal electric flux density and stress on the crack surfaces were zero. In [1], the samples were placed in a Fluorinert-liquid for which $\kappa_a = 1.75\kappa_0$ and κ_0 is the dielectric permittivity of air.

The energy release rate $\mathcal{G}$ and a phase angle ψ were calculated according to the intensity factors for each test. The phase angle ψ is given by

$$\psi = \tan^{-1} \frac{K_{IV}}{K_I} \ . \tag{3}$$

It may be noted that for this case K_{II} was negligible. Two fracture curves, based on these parameters, were obtained as

$$\mathcal{G} = \mathcal{G}_{Ic} \left[1 + \frac{\hat{e}}{\hat{b}} \tan\psi^2 \left(1 + \frac{2\hat{c}}{\hat{e}} \cot\psi \right) \right] \tag{4}$$

$$\mathcal{G} = \mathcal{G}_{IVc} \left[1 + \frac{\hat{b}}{\hat{e}} \cot\psi^2 \left(1 + \frac{2\hat{c}}{\hat{b}} \tan\psi \right) \right] \tag{5}$$

where $\mathcal{G}_{Ic}$ and $\mathcal{G}_{IVc}$ are the critical energy release rates for the first and fourth modes, respectively, determined from averaged tests results, $\hat{b}$, $\hat{c}$ and $\hat{e}$ depend on material properties. The first curve in eq. (4) was fitted for experimental results in which K_I was nearly constant; these results were obtained for the applied electric field $E \geq 0$. The second curve in eq. (5) was fitted for negative electric fields; here K_{IV} was nearly constant. Good agreement was observed between the experimental results and both curves.

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