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CRACK PROPAGATION SIMULATIONS BY A COMBINED FE/BE APPROACH WITH AN AUTOMATIC BE DOMAIN EXTENSION

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Abstract

The accurate and efficient simulation of 3D crack propagation is a challenging task for numerical software packages. While due to their nature Boundary Element Methods (BEM) are very suitable for this issue but lead to high computational costs, volume oriented techniques like the Finite Element Method (FEM) are numerically more efficient but face problems resulting from the domain discretization. Hence, the optimum choice is a combination of both methods.

Here a combined FE/BE approach for the simulation of 3D fatigue crack growth within the framework of linear elastic fracture mechanics is presented. While the major part of the structure is discretized with finite elements, small domains containing arbitrarily shaped 3D cracks are discretized with boundary elements. Due to the nonlinear nature of crack propagation an incremental procedure is required. In each loop (i) the state of stress along the crack front must be determined, (ii) the crack propagation has to be predicted and (iii) the simulation model must be updated. For the simulation of industrial applications special effort must be spent in reducing the computation time.

For the stress analysis the advantageous combination of both numerical techniques is applied [1,2,3]. By means of the Symmetric Galerkin Boundary Element Method (SGBEM) a stiffness formulation for the BE subdomain is obtained: ${}^{\mathrm{B}}\mathbf{K}\;{}^{\mathrm{B}}\mathbf{u} = {}^{\mathrm{B}}\mathbf{f}$. Matrix ${}^{\mathrm{B}}\mathbf{K}$ has the same properties as the stiffness matrix of a single finite element, namely symmetry and positive (semi-)definiteness. As a consequence it can be assembled to the global FE system without destroying its convenient properties and highly efficient solvers can be applied.

The crack growth prediction is based on the results of the BE domain which are approximated only on the boundary but not in the interior. As a consequence the asymptotical singular stress field in the vicinity of the crack front can be captured well. Based on this interior stress field very accurate stress intensity factors (SIF) and T-stresses are computed by an optimized extrapolation and regression technique. By evaluating a 3D crack growth criterion which defines the crack extension and kink angle for every point along the crack front a new front is obtained. Since the state of stress changes between two discrete crack fronts each predicted front is corrected in succeeding increments [4]. As crack propagation occurs only in the BE domain the model update is reduced to the modification of a boundary mesh which is by far less complicated compared to the modification of a volume discretization.

Within each increment the stress analysis – and especially the computation of the stiffness formulation for the BE domain – is the most time consuming part. Therefore it is accelerated by several approaches side by side: (i) By re-using integral contributions to the system matrix which remain constant during crack propagation the integration amount is reduced. (ii) The computation of the 4D integrals and the elimination of boundary element traction variables are parallelized within the SGBEM procedure. Due to the process structure this scales almost linearly with the number of processors. (iii) Since the geometry is hardly changed by corrector steps the stress analysis in the subsequent increment is carried out as a submodel analysis only for the BE domain using so called fast formulations of the collocation technique. Here the tractions in the interface are prescribed as boundary conditions. A submodel analysis is accepted as valid if the work performed by the tractions with the displacements in each boundary element remains within a user-specified tolerance compared to the results of the FE/BE combined model. (iv) It is clear that the smaller the BE domain the faster the computation of its stiffness formulation. But this contradicts with an automated crack propagation analysis as the situation can arise that the crack front reaches or gets close to the FE/BE interface. In order to avoid this and resulting negative effects on the interior stress field we are currently working on an automatic extension of the BE domain. Results will be presented at the conference.

Figure 1a displays the FE/BE combined model of a wheel carrier. Red, gold and green domains are modelled with finite elements, the blue region is discretized with boundary elements and contains an fictitious initial crack. Figure 1b shows predicted and corrected crack fronts after 103 increments. Wall clock times required for the computation of the BE stiffness formulation in the first increment depending on the number of processors are visualized in Figure 1c. The wall clock times required during the whole simulation are plotted in Figure 1d and underline the efficiency of submodel analyses.

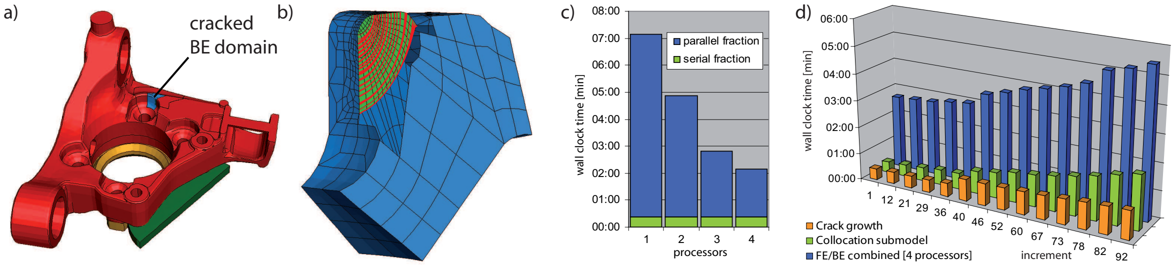

Figure 1: a) combined FE/BE model of a wheel carrier; b) simulated crack fronts; c) wall clock times in the first increment for computating the BE stiffness; d) wall clock times during the simulation

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