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Wave-based numerical methods for acoustics

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ABSTRACT

Recent developments in wave-based numerical methods are reviewed in application to problems in acoustics where small perturbations of pressure and velocity propagate on a steady compressible mean flow. In the most general case, such wave fields are represented by the solution of the linearized Euler equations. In the case of steady time-harmonic solutions in a homogeneous medium and in the absence of mean flow, these reduce to the solution of the Helmholtz equation. When irrotational mean flow is present, they can be represented by a convected Helmholtz-like equation. When rotational mean flow effects are significant, the time harmonic, coupled, first order linearized Euler equations must be solved. Wave-based numerical methods have been applied to all three categories of problem.

The solution of the time-harmonic acoustic equations, particularly for large three-dimensional domains, is computationally challenging. When traditional finite element methods are used with polynomial basis functions within each element, approximability arguments alone require the use of many nodes per wavelength to resolve the resulting spatially harmonic solutions. It is moreover well known that these requirements are exacerbated when the computational domain spans many wavelengths of the solution. In such cases the 'pollution' effect [1] means that the global error increases with frequency, irrespective of the number of nodes per wavelength that are used to resolve the solution. This effect is particularly acute for problems which involve exterior scattering by objects whose geometric lengthscale is large compared to a typical wavelength. A problem this type which is of particular interest to the authors is acoustic radiation from aero-engine nacelles where the length scale of the acoustic disturbance at peak frequencies is an order of magnitude smaller than the diameter of the nacelle. Another problem which exhibits the same disparity of lengthscales and where wave-based methods have been recently been applied is in the calculation of Head Related Transfer Functions for the human head and torso. Here a similar relationship holds between the wavelength of the disturbance and the dimensions of the torso over much of the audible range. To resolve either of these problems in three dimensions by using conventional numerical methods requires many millions of node or grid points.

The use of non-polynomial, wave-like bases to represent such solutions more effectively with fewer degrees of freedom, can be traced back to infinite element schemes developed over several decades, in which a single outwardly propagating wave direction is used [2,3]. Such methods are now routinely implemented in a number of commercial codes. More recently, the Partition of Unity method [4,5,6] has

been applied to a more general class of problem where no *a priori* wave direction is known. Here a continuous wave-like trial solution is generated from a conventional Finite Element mesh by multiplying the shape functions by a discrete set of wave solutions.

A number of discontinuous formulations have also been explored. In such methods, continuity at element boundaries must be imposed on wavelike trial solutions defined within each element [7,8]. In the case of the Helmholtz problem, such methods can also be regarded as Trefftz solutions [9], and have been shown to embrace also the more esoteric Ultra-Weak Variational Approach [10].

All of the above methods suffer from poor conditioning as the number of wave directions increases. The extent to which this is an impediment to their practical implementation is significant in assessing the utility of each method, as are potential pre-conditioning strategies to reduce condition number.

While most of the methods noted above, have been developed for the homogeneous Helmholtz problem in the absence of flow, some have been applied also to the case with non-zero mean flow [8]. Particular attention will be given in the current review to the effectiveness of these formulations for the flow case.

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