Linear stability analysis of an alternating magnetic field driven flow in a spinning container

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ABSTRACT

We present a numerical analysis of the free surface liquid metal flow driven by an alternating (AC) magnetic field in a spinning cylindrical container. The axysimmetric flow structure is analyzed for various values of the magnetohydrodynamic interaction parameter N and the Ekman number E. The governing hydrodynamic equations are solved by a spectral collocation method. The alternating magnetic field distribution is found by a boundary-integral method. The electromagnetic and hydrodynamic fields are fully coupled via the shape of the liquid free surface. The upper free boundary was found simultaneously with the flow by a Newton method. It is found that in all considered parameter ranges the flow contains four main toroidal eddies. This is caused by the non-uniformity of the magnetic field near the edges of the liquid volume. The interaction parameter N controls the intensity of the flow. The additional container spinning leads to a deformation of the flow structure. At Ekman number $E < 1 \times 10^{-2}$ the meridional flow is reduced. The secondary azimuthal flow has its maximum in the Ekman number range of $E \sim 10^{-3} - 10^{-2}$, at smaller Ekman number the azimuthal flow is suppressed too.

The three-dimensional stability analysis of the flow showed that the spinning leads mainly to a destabilization of the base flow. Only at very small Ekman numbers E the flow in the spinning container is more stable than in the non-spinning case. The instability at large Ekman numbers is of oscillatory type and the most unstable azimuthal wave number is m = 3. At smaller Ekman numbers the azimuthal wave number increased to m = 5, m = 6, etc. At $E < 2.1 \times 10^{-4}$ the most unstable wave number is m = 16. Except the narrow Ekman number range of $1.935 \times 10^{-2} < E < 2.376 \times 10^{-2}$ where the instability is of oscillatory type, at all other values of Ekman number $E < 4.6136 \times 10^{-2}$ the instability is of steady type.