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SPLIT OF TERRITORIES FOR OPTIMUM-SHAPE DESIGN IN AERODYNAMICS AND COUPLED DISCIPLINES

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ABSTRACT

When devising a numerical shape–optimization method in the context of a complex engineering situation, the practitioner is faced to an acute difficulty related to the participation, in a realistic formulation, of several relevant physical criteria originating from several disciplines. Perhaps the most commonly–used treatment of multi–criterion problems is the penalization approach in which one minimizes a single functional agglomerating all the criteria weighted by penalty constants. The method is computationally economical, but evidently, the resulting solution depends on the penalty constants whose adjustment is usually made with a fair amount of arbitrariness. Alternately, at the other extreme in computational cost involved, when feasible, identifying Pareto fronts made of non–dominated design points, has the great merit of providing the designer with a very rich information about the system supporting the decision making. However the corresponding implementation requires a very large number of simultaneous evaluations of several functionals.

A treatment of multi–criterion problems that removes the question of adjusting the penalty constants, and that is computationally more economical than identifying the Pareto equilibrium front, is to seek a pseudo–optimal solution as the equilibrium point of a simulated

considered as the strategy or territory of a given functional. Nash or Stackelberg games are usually considered [1]-[2]. Of course, the adopted definition of the splitting also introduces a bias, but we momentarily put aside this question. Examples of successful concurrent optimizations realized numerically by such dynamic games can be found for example in [3] and [4]. The full paper will reproduce some of these successful experiments for purpose of demonstration, but also to point out the difficulty to define in general a proper split of design variables in adequacy with the physics of the problem.

A theory has recently been developed [5] for the treatment of cases in which a primary and a secondary criteria are to be optimized. The absolute optimum of the primary criterion is first identified, presumably numerically. Then, a secondary criterion is improved in a virtual Nash game, in which the design variables have been split, according to the diagonalization of a projected Hessian, and assigned to the two virtual players, in a way that is devised to cause the least possible degradation to the primary criterion from its absolute optimum. Additionally, the theory puts in evidence the existence of a continuum of Nash equilibrium points originating from the initial absolute optimum of the primary criterion considered alone.

An example of application of this methodology of split will be extracted from the recent thesis of B. Abou El Majd [6]. There, a wing shape has been optimized first w.r.t. an aerodynamic criterion; then a structural criterion has been improved in a second phase of optimization through a virtual Nash dynamic game. Despite the fragility of the aerodynamic criterion in this difficult exercise, an aerodynamically-relevant result has been achieved, and the physical pertinence of the mathematical split, although automatic, is evident.

Thirdly, we plan to illustrate the same procedure of split of territories applied to an airfoil optimization w.r.t. aerodynamics (primary criterion governed by the compressible Euler equations) and a stealth criterion (secondary criterion governed by the Maxwell equations). In this case, no intuitive arguments are available to guide an a priori pertinent geometrical split of territories.

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