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EXISTENCE AND UNIQUENESS OF SOLUTIONS IN NONASSOCIATED MOHR-COULOMB ELASTOPLASTICITY

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ABSTRACT

In 1902 Jacques Hadamard [1] formulated three conditions that mathematical models of physical phenomena should satisfy, namely that:

(a) A solution exists
(b) The solution is unique
(c) The solution depends continuously on the problem data

Problems involving models that satisfy all of these conditions are termed well-posed. Otherwise, if one or more conditions are violated, the problem is said to be ill-posed.

The meaning of condition (a) is quite clear. Conditions (b) and (c) are often intimately linked and can for many practical purposes be coalesced into a single condition. In finite element analysis "non-unique" solutions are often generated by altering the problem data slightly, for example through introduction of random imperfections.

It is relatively well known (though not always appreciated to its full extent) that standard elastoplastic constitutive models may violate conditions (b) and (c) if the flow rule is not associated. Indeed, since the pioneering work of Rice [2], the ensuing ill-posedness has often been given a physical interpretation, particularly in terms of discontinuous bifurcations (strain localization, shear banding, etc) but also in terms of other experimentally observed bifurcation phenomena (see e.g. Borja [3]).

On the other hand, the issue of existence has been given much less attention. In fact, to the authors' knowledge, it has not been dealt with at all in the mainstream literature. However, among computational modelers it is widely recognized that constitutive models involving nonassociated flow rules give rise to a variety of "numerical problems", the most common one being a marked difficulty in achieving convergence of the iterative procedure used for solving the nonlinear finite element equations. For the classical linear elastic/perfectly plastic Mohr-Coulomb model, these difficulties typically become more pronounced as the difference between the friction and dilation angle increases (for a fixed mesh density) or as the mesh density increases (for a fixed

difference between the friction and dilation angle). Such observations have been made for a wide variety of solution techniques and procedures (implicit, explicit, different finite elements, etc.).

There is thus a significant body of circumstantial evidence that all three of Hadamard's conditions of well-posedness may be violated as a result of nonassociated flow rules. In the paper we attempt to add to this evidence by systematically studying the behaviour of a number of selected problems for varying mesh densities and degrees of nonassociativity. Secondly, we show how nonassociated flow rules (in the conventional sense) in fact can be avoided by employing a recently proposed variational framework of elastoplastic constitutive modeling [4,5].

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