

Geometric decompositions and bases for spaces of piecewise polynomial differential forms

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Key Words: *finite element, geometric decomposition, basis*

ABSTRACT

We study the two primary families of spaces of piecewise polynomial differential forms with respect to a simplicial mesh in R^n . These spaces are generalizations of the classical finite element spaces for vector fields, frequently referred to as Raviart–Thomas spaces, Nedeléc spaces, or Brezzi–Douglas–Marini spaces. The purpose of this work is to give geometric decompositions of these spaces, generalizing the Bernstein decomposition of ordinary polynomials defined on a simplex in R^n . By treating both families of finite elements together, and adopting the framework of differential forms, we are able to show the close connection of these two families and also how these decompositions lead to explicit bases for these two families of spaces of piecewise polynomial differential forms for arbitrary polynomial degree, arbitrary order of the forms, and any space dimension.

REFERENCES

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