8th. World Congress on Computational Mechanics (WCCM8)
5th European Congress on Computational Methods in Applied Sciences and Engineeering (ECCOMAS 2008)
June 30 –July 5, 2008
Venice, Italy

MULTIGRID AND STOCHASTIC SPARSE-GRID TECHNIQUES FOR TIME-DEPENDENT CONTROL PROBLEMS WITH RANDOM INPUTS

*** Alfio Borzì[1] and Greg von Winckel[2]**

[1] Institute for Mathematics and Scientific Computing Karl-Franzens-University, Heinrichstr. 36, 8010 Graz, Austria
alfio.borzi@uni-graz.at
http://www.uni-graz.at/imawww/borzi/index.html

[2] Institute for Mathematics and Scientific Computing Karl-Franzens-University, Heinrichstr. 36, 8010 Graz, Austria
alfio.borzi@uni-graz.at
http://www.uni-graz.at/imawww/borzi/index.html

Key Words: *Stochastic PDE, optimal control, multigrid methods, sparse grids.*

ABSTRACT

The influence of randomness or uncertainty of the input data on the control provided by the optimal control theory framework is investigated considering a class of nonlinear parabolic optimal control problems. The focus is on governing equations where diffusion and reaction parameters are represented by random fields.

For the purpose of this investigation, parabolic optimal control problems with random input data are formulated and space-time multigrid methods combined with stochastic sparse-grid collocation techniques are implemented to robustly and efficiently span the physical and the stochastic spaces where these optimization problems are defined. Sparse-grid collocation and twogrid Fourier analysis are used to provide estimates of convergence of the resulting algorithms. These estimates are in agreement with results of experiments. Results of computation of stochastic optimal control solutions and investigation of robustness in the present framework are presented.

REFERENCES

[1] A. Borzì. "Space-time multigrid methods for solving unsteady optimal control problems". Chapter 5 in L.T. Biegler, O. Ghattas, M. Heinkenschloss, D. Keyes and B. van Bloemen Waanders (Eds.), *Real-Time PDE-Constrained Optimization, Computational Science and Engineering*, Vol. **3**, SIAM, Philadelphia, 2007.

[2] F. Nobile, R. Tempone and C. Webster. "A sparse grid stochastic collocation method for elliptic partial differential equations with random input data", Report TRITA-NA 2007:7, NADA, Royal Institute of Technology, Stockholm, Sweden.

[3] D. Xiu and J.S. Hesthaven, "High-order collocation methods for differential equations with random inputs", SIAM J. Sci. Comp., 27(3) (2005), pp. 1118–1139.