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DYNAMIC ANALYSIS OF INFLATABLE MEMBRANES COUPLED WITH ENCLOSED FLUIDS

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ABSTRACT

The interaction of enclosed fluid and surrounding membranes can be tackled by a direct modelling and discretization of solid and fluid domains, however at the price of high computational cost. Alternatively, for fluid-structure interaction problems whose fluid behaviour is not the main interest, the discretization of the fluid domain is avoidable by the definition of deformation dependent forces, always acting perpendicular to the membrane surfaces. Moreover, the fluid pressure is varied along the change of the enclosed volume. The relationship between the deformation dependent pressure and the volume variation can be described by a suitable state equation.

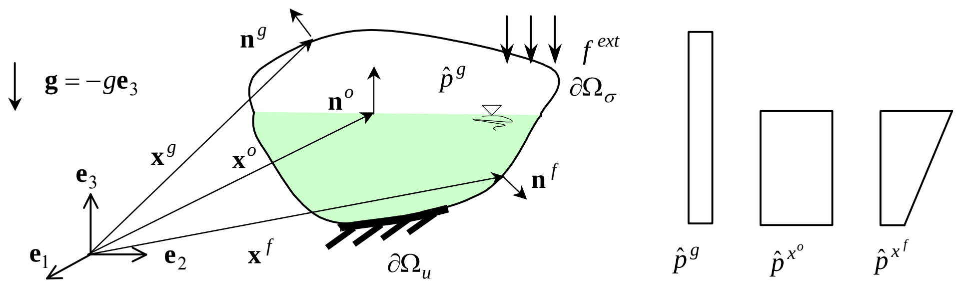

Figure 1: Membrane filled with incompressible fluid and gas.

This contribution extends the method from a static analysis[1] to the dynamic response of inflatable membrane structures with the coupling stiffness between enclosed fluid and surrounding membranes. The result of this study emphasizes that the influence of enclosed volume-pressure coupling, which is usually neglected, to the total stiffness of inflatable membranes is indispensable in case of highly pressurized gas and high density fluids. Moreover, the rate of convergence is improved significantly by the inclusion of enclosed volume terms. Besides, a wrinkling model based on the plasticity analogy approach[3] is successfully incorporated to remove the artificial compressive stiffness from membrane elements during wrinkling.

For an incompressible fluid with free fluid surface and overpressure gas, as can be seen in Fig.1, the pressure at the wetted surface below the free fluid surface is stated as:

$$\hat{p}^f = -\hat{p}^{x^o} + \hat{p}^{x^f} + \hat{p}^g \ ; \ \hat{p}^{x^o} = \rho\boldsymbol{g}\cdot\boldsymbol{x}^o, \hat{p}^{x^f} = \rho\boldsymbol{g}\cdot\boldsymbol{x}^f \ , \ \hat{p}^g = \frac{P_0V_0^k}{V^k}. \tag{1}$$

Referred to the reference level, $\hat{p}^{x^o}$ is the pressure at the free fluid surface and $\hat{p}^{x^f}$ is the pressure at a point in the fluid domain. Subtraction of them yields the hydrostatic pressure. $\hat{p}^g$ designates the gas pressure and $\boldsymbol{n}^o$ is outward unit normal vectors at the free fluid surface while $\boldsymbol{n}^g$ and $\boldsymbol{n}^f$ are ones above and below the free fluid surface, respectively. Total virtual work is composed of inertial, internal, external and follower force contributions as

$$-\delta W = -\delta W_{inert} - \delta W_{\text{int}} - \delta W_{ext} - \delta W_{fol} = 0, \tag{2}$$

where the follower force part of the enclosed gas δW_{fol}^g and fluid δW_{fol}^f are stated by

$$\delta W_{fol} = \int_{\theta^1}\int_{\theta^2} \hat{p}\boldsymbol{n}^* \cdot \delta\boldsymbol{u}\, d\theta^1 d\theta^2 = \int_{\theta^1}\int_{\theta^2} \hat{p}^g \boldsymbol{n}^{*g} \cdot \delta\boldsymbol{u}\, d\theta^1 d\theta^2 + \int_{\theta^1}\int_{\theta^2} \hat{p}^f \boldsymbol{n}^{*f} \cdot \delta\boldsymbol{u}\, d\theta^1 d\theta^2 ;\ \boldsymbol{n}^* = \boldsymbol{x}_{,\theta^1} \times \boldsymbol{x}_{,\theta^2}, \tag{3}$$

which $\mathbf{n}^*$ is the nonnormalized surface normal vector. According to the generalized alfa method[4], the equation of motion is expressed by

$$\boldsymbol{M}\ddot{\boldsymbol{u}}_{n+1-\alpha_m} + \boldsymbol{C}\dot{\boldsymbol{u}}_{n+1-\alpha_f} + \boldsymbol{f}^{\text{int}}(\boldsymbol{u}_{n+1-\alpha_f}) - \boldsymbol{f}^{ext}(\boldsymbol{u}_{n+1-\alpha_f}) - \boldsymbol{f}^{fol}(\boldsymbol{u}_{n+1-\alpha_f}) = \boldsymbol{0}, \tag{4}$$

where $\ddot{\boldsymbol{u}}_{n+1-\alpha_m} = (1-\alpha_m)\ddot{\boldsymbol{u}}_{n+1} + \alpha_m \ddot{\boldsymbol{u}}_n$; $\dot{\boldsymbol{u}}_{n+1-\alpha_f} = (1-\alpha_f)\dot{\boldsymbol{u}}_{n+1} + \alpha_f \dot{\boldsymbol{u}}_n$; $\boldsymbol{u}_{n+1-\alpha_f} = (1-\alpha_f)\boldsymbol{u}_{n+1} + \alpha_f \boldsymbol{u}_n$; $\boldsymbol{f}^{\text{int},ext,fol}(\boldsymbol{u}_{n+1-\alpha_f}) = (1-\alpha_f)\boldsymbol{f}_{n+1}^{\text{int},ext,fol} + \alpha_f \boldsymbol{f}_n^{\text{int},ext,fol}$ with the shift parameters α_m and α_f. The residual at the unknown displacement $\boldsymbol{u}_{n+1}$ can be written as

$$\boldsymbol{G}(\boldsymbol{u}_{n+1}) = \underbrace{\frac{1-\alpha_m}{\beta\Delta t^2}\boldsymbol{M}\boldsymbol{u}_{n+1} + \frac{(1-\alpha_f)\gamma}{\beta\Delta t}\boldsymbol{C}\boldsymbol{u}_{n+1} - \boldsymbol{h}(\boldsymbol{u}_n, \dot{\boldsymbol{u}}_n, \ddot{\boldsymbol{u}}_n)}_{\boldsymbol{f}^{inert}} + \boldsymbol{f}^{\text{int}}(\boldsymbol{u}_{n+1-\alpha_f}) - \boldsymbol{f}^{ext}(\boldsymbol{u}_{n+1-\alpha_f}) - \boldsymbol{f}^{fol}(\boldsymbol{u}_{n+1-\alpha_f}), \tag{5}$$

where the time step $\Delta t = t_{n+1} - t_n$ and $\boldsymbol{h}(\boldsymbol{u}_n, \dot{\boldsymbol{u}}_n, \ddot{\boldsymbol{u}}_n)$ is the history term[4]. After linearization, the follower force stiffness is inserted in the effective iterative structure equation for a single chamber along the line of the Newton-Raphson technique at iteration step k as

$$\boldsymbol{G}(\boldsymbol{u}_{n+1}^{k+1}) \approx \boldsymbol{G}(\boldsymbol{u}_{n+1}^k) + \frac{\partial \boldsymbol{G}(\boldsymbol{u}_{n+1}^k)}{\partial \boldsymbol{u}_{n+1}}\Delta\boldsymbol{u} = \boldsymbol{0} \Rightarrow \boldsymbol{K}_{eff}^k \Delta\boldsymbol{u} = -\boldsymbol{G}(\boldsymbol{u}_{n+1}^k) = -\boldsymbol{f}^{inert} - \boldsymbol{f}^{\text{int}} + \boldsymbol{f}^{ext} + \boldsymbol{f}^{fol}, \tag{6}$$

$$\text{Where } \boldsymbol{K}_{eff}^k = \underbrace{\frac{1-\alpha_m}{\beta\Delta t^2}\boldsymbol{M} + \frac{(1-\alpha_f)\gamma}{\beta\Delta t}\boldsymbol{C}}_{\boldsymbol{K}_{inert}^k} + \underbrace{\frac{\partial \boldsymbol{f}^{\text{int}}(\boldsymbol{u}_{n+1-\alpha_f}(\boldsymbol{u}_{n+1}))}{\partial \boldsymbol{u}_{n+1}}}_{\boldsymbol{K}_E^k + \boldsymbol{K}_G^k} - \underbrace{\frac{\partial \boldsymbol{f}^{fol}(\boldsymbol{u}_{n+1-\alpha_f}(\boldsymbol{u}_{n+1}))}{\partial \boldsymbol{u}_{n+1}}}_{\boldsymbol{K}_L^k}, \tag{7}$$

$$with\ \boldsymbol{K}_L^k = \frac{\partial \boldsymbol{f}^{fol}(\boldsymbol{u}_{n+1-\alpha_f}(\boldsymbol{u}_{n+1}))}{\partial \boldsymbol{u}_{n+1}} = \begin{bmatrix} \boldsymbol{K}_{0L}^k + \beta^0(\boldsymbol{b}\otimes\boldsymbol{b} + \boldsymbol{b}\otimes\boldsymbol{b}) \\ + \beta^1(\boldsymbol{a}\otimes\boldsymbol{a} + \boldsymbol{b}\otimes\boldsymbol{b} + \boldsymbol{a}\otimes\boldsymbol{b} + \boldsymbol{b}\otimes\boldsymbol{a}) \end{bmatrix}. \tag{8}$$

where $\boldsymbol{K}_E^k, \boldsymbol{K}_G^k$ stand for the elastic and geometric stiffness, accordingly. The symmetric load stiffness matrix $\boldsymbol{K}_L^k$ reflects the conservativeness of the described problem with follower forces abiding by special boundary conditions[2]. Obviously, the coupling of the gas volume and the fluid depth variation is expressed in term $\beta^1(\boldsymbol{a}\otimes\boldsymbol{b} + \boldsymbol{b}\otimes\boldsymbol{a})$ where $\boldsymbol{a}$ and $\boldsymbol{b}$ are update vectors[2] in the general form for gas and fluid, respectively. β^0 and β^1 stand for the gradients of the gas pressure and the fluid pressure, correspondingly.

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