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# EDGE-BY-EDGE IMPLEMENTATION OF RESIDUAL-BASED VARIATIONAL MULTISCALE METHOD

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## ABSTRACT

The Residual-Based Variational Multiscale (RB-VMS for short) is emerging as a new concept in Large-Eddy Simulation (LES) of turbulent flows. It was introduced in Calo [1], who studied bypass transition on a zero-pressure gradient flat plate. Calo also have shown an implementation of RB-VMS on a second-order finite volume code and described how to derive a weak formulation that yields an equivalent discrete equation system. Gravemeier et al [2] studied turbulent flows in a channel at Re=180 with a RB-VMS finite element method showing the importance of higher-order polynomial approximations. Bazilevs et al [3] presented a LES-type VMS theory of turbulence and tested it on forced homogeneous turbulence, isotropic turbulence and turbulent homogeneous channel flows, stressing the superior quality of NURBS elements with respect to classical finite elements. Akkerman et al [4] examined the role of continuity in NURBS based computation of turbulent flows with RB-VMS applied to the advective-form of the governing equations.

Edge-based data structures have been introduced in the finite element context to speed-up explicit compressible flow simulations (see Lohner [5]). It has been shown by Ribeiro and Coutinho [6] that, for unstructured grids, edge-based data structures decrease the number of floating point operations and indirect addressing in matrix-vector products needed in Krylov space solvers and diminish the storage area to hold Jacobians compared to element and pointwise data structures, particularly for problems involving many degrees of freedom. The construction of edge operations are completely algebraic, based on the concept of disassembling element operators, regardless of the particular underlying finite element formulation, thus providing a fast platform for simulation of complex problems.

In this work we extend our edge-based stabilized finite element incompressible flow solver to turbulence modeling with RB-VMS. The main characteristics of our basic solver are: SUPG, Pressure-Stabilizing/Petrov-Galerkin (PSPG) and Least-Squares Incompressibility Constraint (LSIC) stabilized finite element formulation; implicit time

marching scheme with adaptive time stepping control; advanced Inexact Newton solvers; edge-based data structures to save memory and improve performance; support to message passing and shared memory parallel programming models; LES by Smagorinsky's model; Volume-of-Fluid (VOF) extensions to track the evolving free-surfaces [7]. We show how to incorporate RB-VMS, using the advective-form of the convection term of the Navier-Stokes equations [4], as a straightforward extension of standard stabilized methods with a modified advective velocity. This requires minimum modification of the existing highly optimized code. We solved the incompressible flow past a circular cylinder at Re=100 and compared RB-VMS with standard stabilized formulations. We have shown that RB-VMS presented similar accuracy and performance as SUPG/PSPG/LSIC in this laminar test case. We then compare our results in the lid-driven cubic cavity at Re=10,000 problem where recent LES, DNS and experimental results are available [8]. We noticed that the present implementation was able to capture most of the relevant turbulent flow features with reasonable accuracy when compared with highly resolved numerical simulations and experimental data. Thus, our initial tests in laminar and turbulent flows have shown promising results.

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