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Well-balanced high-order finite volume schemes for shallow water equations with topography and dry areas

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ABSTRACT

Shallow water equations are widely used in ocean and hydraulic engineering to model flows in rivers, reservoirs or coastal areas, among others applications. In the form considered in this paper, they constitute a hyperbolic system of conservation laws with a source term due to the bottom topography.

In recent years, there has been increasing interest concerning the design of high-order numerical schemes for shallow water equations. These kind of schemes compute solutions with high-order accuracy (both in space and time) in the regions where the solutions are smooth, while at the same time shock discontinuities are properly captured. Usually, these schemes are based on high-order reconstructions of numerical fluxes or states. However, when a source term is present the schemes must also satisfy a balance between the flux and the source terms, in order to properly compute stationary or almost stationary solutions. This property is known as *well-balancing*, and it is currently an active subject of research.

In [1], a high-order well-balanced finite volume scheme was developed in a general *nonconservative* framework. The scheme uses high-order reconstructions of states and it is based on the concept of *generalized Roe schemes*. In particular, it was successfully applied to shallow water equations with bottom topography, using fifth-order WENO reconstructions in space and a third-order TVD Runge-Kutta scheme to advance in time.

An important difficulty arising in the simulation of free surface flows is the appearance of dry areas, due to the initial conditions or as a result of the motion of the fluid. Examples are numerous: flood waves, dambreaks, breaking of waves on beaches, etc. If no modifications are made, standard numerical schemes may fail in the presence of wet/dry situations, producing spurious results. Several methods can be found in the literature which overcome this problem.

When applied to shallow water equations, the generalized Roe schemes lose their well-balance properties in the presence of wet/dry transitions. Moreover, they may produce negative values of the thickness of the water layer in the proximities of the wet/dry front. Recently, a new technique for treating wet/dry fronts in the context of Roe schemes has been presented in [2]. It consists in replacing, at the intercells

where a wet/dry transition has been detected, the corresponding linear Riemann problem by an adequate nonlinear one.

The goal of the present work is to properly combine the scheme developed in [1] with the treatment of wet/dry fronts introduced in [2]. This is by no means an easy task, as many difficulties appear. In particular, the numerical fluxes must be modified accordingly to the kind of wet/dry transition found. Moreover, the variables to be reconstructed have to be properly chosen in order to maintain the well-balance property of the scheme and, at the same time, to preserve the positivity of the water height. In particular, the hyperbolic reconstruction method introduced in [3] has been considered, as we have proved that it assures the positivity of the reconstruction of the water height at each computational cell. If instead polynomial reconstructions are used, they may introduce oscillations leading to the appearance of negative values of the water height.

Finally, a recently developed extension of the scheme to two-dimensional shallow flows, based on the bi-hyperbolic reconstruction technique introduced in [4], will be also presented.

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