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CONTRASTING HIGH-RESOLUTION SCHEMES FOR GAS DYNAMICS TEST PROBLEMS

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ABSTRACT

A popular class of high-resolution finite-difference methods is based on solving equations of motion in the strong conservation form and applying non-linear flux correction procedures to enforce the monotonicity, or Total Variation Diminishing (TVD), property in the solution. TVD schemes have become particular popular in computations of flows involving high gradients, since they are based on a fixed grid computational stencil, are easy to implement, and are robust in many engineering and physical applications, except for very strong shocks. However, many popular TVD methods are based on standard linear finite-difference schemes, and the application of flux-limiting procedures to them brings about a notable numerical dissipation to the solution, which smears the linear flow field. By using higher-order flux reconstruction procedures, such as higher-order versions of Weighted Essentially Non- Oscillatory reconstruction (WENO), Discontinuous Galerkin (DG), and Arbitrary Derivative Riemann procedures (ADER), the linear wave properties of the original "low-order" schemes can be improved. The cost of using high-order schemes is generally accepted by the computational community for those problems where the standard second-order methods require too fine grid to resolve short wavelengths without considerable amplitude and phase errors. In general, it poses a question whether second-order schemes are at all viable to use for problems which are sensitive to linear wave propagation properties. One motivation of this paper is to demonstrate that second-order methods can indeed be very efficient for flow problems ranging from linear acoustic flows to very strong shocks.

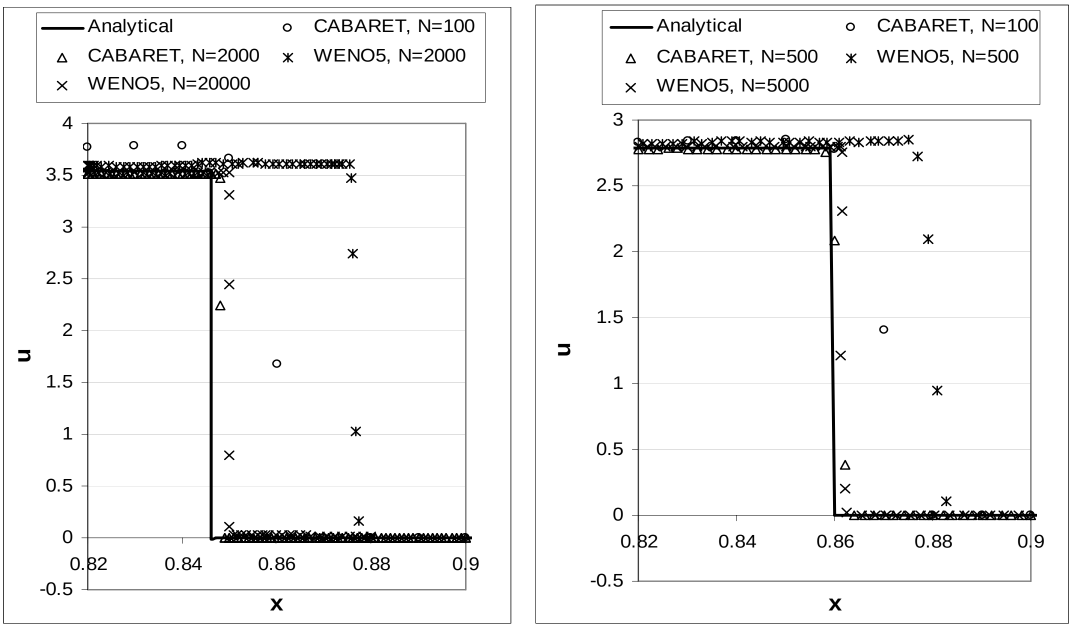
To illustrate the point we compare several high-resolution schemes for gas dynamics test problems. One of them is the novel CABARET scheme, which was introduced in [1,2]. The CABARET is second-order, non-dissipative and has a very compact computational stencil defined within one computational cell in space and time. For shock-capturing it can be equipped with the non-linear correction procedure which preserves the maximum principle without bringing about a significant numerical diffusion.

NUMERICAL EXAMPLE

To illustrate the shock-capturing capability of the CABARET scheme, the 1-D shock-tube problems from [3] is considered. In comparison to standard “Sod Test Problems” the ideal gas has a higher density and pressure ratio across the initial discontinuity:

$$(\rho,u,p)=\begin{cases}(10^3,0,10^3) & \text{if } 0\le x<0.3\\ (1,0,1) & \text{if } 0.3<x\le 1\end{cases}\text{(1)};\quad (\rho,u,p)=\begin{cases}(10^4,0,10^4) & \text{if } 0\le x<0.3\\ (1,0,1) & \text{if } 0.3<x\le 1\end{cases}\text{(2)}.$$

Popular high-resolution long-stencil schemes, such as the fifth-order Weighted Essentially Non-Oscillatory (WENO5) scheme, have a very slow convergence of the velocity across the shock. In contrast to those, the CABARET scheme remains efficient on grids, which are in order of magnitude coarser (Fig.1).



(a) **(b)**

Figure 1. Velocity profile across the shock: comparing the CABARET solution with the fifth-order WENO scheme (WENO5) [3], (a) – Problem 1, t=0.15 and (b) – Problem 2, t=0.12

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