8th. World Congress on Computational Mechanics (WCCM8)
5th. European Congress on Computational Methods in Applied Sciences and Engineering (ECCOMAS 2008)
June 30 – July 5, 2008
Venice, Italy

A NEW FAST, ACCURATE AND NON-OSCILLATORY NUMERICAL APPROACH FOR WAVE PROPAGATION PROBLEMS IN SOLIDS

A. Idesman

Department of Mechanical Engineering, Texas Tech University
Box 41021, Lubbock, TX 79409, USA
alexander.idesman@coe.ttu.edu

Key Words: *Wave propagation, FEM, Time integration, Numerical dissipation.*

ABSTRACT

There are the following issues with existing numerical methods for elastodynamics problems: a) due to spurious high-frequency oscillations, the lack of reliable numerical techniques that yield an accurate solution of wave propagation in solids; b) the treatment of error accumulation for long-term integration; c) the selection of an effective numerical method among known ones; d) the increase in accuracy and the reduction of computation time for real-world dynamic problems.

A new numerical approach for computer simulation of the dynamic response of complex structures is suggested. The new technique is very general, and would be of equal value in such diverse applications as: explosions; earthquakes; crashes; dynamics testing of aerospace vehicles, airplanes, bridges and buildings; and many others. The new approach includes a new solution strategy and new second- and high-order accurate time-integration methods for elastodynamics, and resolves the issues listed. The finite element method is used for the space discretization.

The new solution strategy consists of two stages: basic computations and post-processing. Basic computations should be done by a method with zero numerical dissipation (zero artificial viscosity) and allow large high-frequency oscillations. Post-processing will be done by a method with large numerical dissipation for filtering high-frequency oscillations. It is proved that for linear elastodynamics the new strategy yields the most accurate results compared to existing approaches. New fundamental results have been obtained due to the new strategy: e.g., the trapezoidal rule is the best method for basic computations among all second-order implicit methods; in contrast to textbooks on finite elements, for long-term integration, the size of time increments for explicit methods should be much smaller than the stability limit (rather than close to it) and depend on the total number of time increments. The extension of the strategy to non-linear problems is possible in many cases and will be discussed.

For effective implementation of the new solution strategy, new implicit and explicit high-order accurate time-continuous Galerkin (TCG) methods with controllable numerical dissipation are developed (see also [1, 2]). The accuracy of the new TCG methods is higher than the accuracy of known methods at the same number of degrees of freedom. For the selection of the size of time increments at long-term integration, a new a priori error estimator in time is developed.

1-D and 2-D numerical examples (e.g., see Figs. 1, 2) show that the new approach with implicit or explicit TCG methods allows a non-oscillatory solution for wave propagation in solids and reduces computation time by 5-50 times and more in comparison to the time required by the existing second-order methods used in most commercial software. For the first time a reliable, fast, accurate and non-oscillatory solution of wave propagation in solids is possible. In contrast to existing approaches, the new technique does not require any guesswork for the selection of numerical dissipation or artificial viscosity and retains the accuracy of the basic solution at low modes. Based on the new approach, for the first time an accurate solution of high-frequency pulse propagation in the Hopkinson Pressure Bar is calculated (without reliance on assumptions for the correction of oscillatory results).

[1] A. V. Idesman, "A new high-order accurate continuous Galerkin method for linear elastodynamics problems", *Computational Mechanics*, Vol. **40**, pp. 261-279, (2007).

[2] A. V. Idesman, "Solution of linear elastodynamics problems with space-time finite elements on structured and unstructured meshes", *Computer Methods in Applied Mechanics and Engineering*, Vol. **196**, pp. 1787–1815, (2007).

[3] F. Vales, S. Moravka, R. Brepta, and J. Cerv. Wave propagation in a thick cylindrical bar due to longitudinal impact. JSME International Journal, Series A, Vol. **39**, pp. 60-70, (1996).

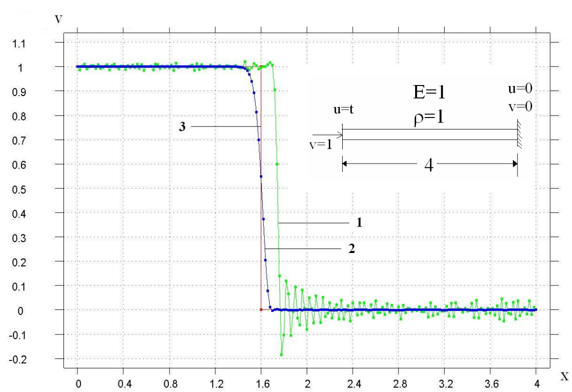

Fig.1 The velocity distribution along the 1-D bar during impact (after 5 wave reflections from the bar ends). Curves 1, 2 and 3 correspond to basic computations at time T=22.26, post-processing (8 time increments) at time T=22.4 and the analytical solution at time T=22.4. A uniform mesh with 200 quadratic 3-node elements is used.

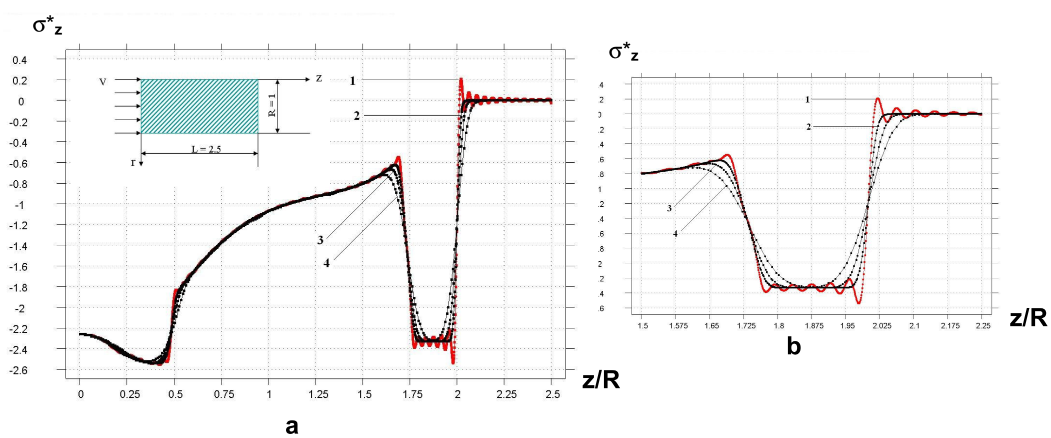

Fig. 2. The distribution of the dimensionless axial stress along the axial coordinate and the fixed radial coordinate *r/R*= 0.05 (R is the radius of the bar, see [3] for the notations) at dimensionless time t = 2. The axisymmetric formulation of the impact problem shown in Fig. 1a is considered. Curve 1 with oscillations due to Gibbs phenomena is the approximation of the analytical solution; see [3]. Curves 2, 3 and 4 correspond to the numerical solutions obtained with the new approach on uniform meshes with 80000, 20000 and 5000 quadratic 9-node elements, respectively. b) is the detailed representation of a) in the range $1.5 < z/R < 2.25$.