

A LEAST-SQUARES APPROXIMATION OF HIGH-DIMENSIONAL UNCERTAIN SYSTEMS

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ABSTRACT

This paper investigates the efficiency of some existing uncertainty propagation schemes for the solution of stochastic partial differential equations (SPDEs) with large number of input uncertain parameters. The uncertainty quantification schemes based on stochastic Galerkin projections, with global or local basis functions, and also sparse grid collocations, in their conventional form, suffer from the so called *curse of dimensionality*: the associated computational cost grows exponentially as a function of the number of random variables defining the underlying probability space of the problem.

In this work, to break the problem of curse of dimensionality, an efficient least-squares scheme is utilized to obtain a low-rank approximation of the solution of an SPDE with high-dimensional random input data. It will be shown that, in theory, the computational cost of the proposed algorithm grows linearly with respect to the dimension of the underlying probability space of the system. Different aspects of the proposed methodology are clarified through its application to a convection-diffusion problem.