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PRACTICAL SHAPE OPTIMIZATION METHOD FOR DESIGN OF THREE DIMENSIONAL STRUCTURE

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ABSTRACT

This paper presents a practical shape optimization method for the design of a three dimensional structure (i.e., solid structure) which has a lot of design degrees of freedom. We have developed a non-parametric shape optimization method, which we call “traction method”, and applied it to various shape design problems of 2D and 3D continua including plate and shell structures in the previous works. In this paper we first apply the traction method to the shape optimization of Neumann boundary on a solid structure under the assumption that the Neumann boundary is allowed to vary. This problem is important to reduce the applied loads in the structural design.

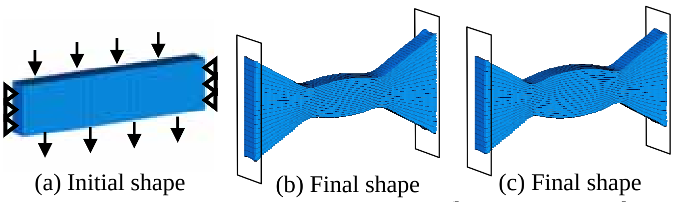
With the traction method we can obtain the optimum shape without shape design parameterization. This method is a gradient method in Hilbert space. A negative shape sensitivity function**,** which is derived usigng the material derivative method, acts as an external force in the normal direction to the design boundaries. The resultant displacement field represents the amount of domain variation (i.e., design velocity field). The velocity field thus obtained is then added to the original shape. By repeating this procedure alternately, the optimal shape is obtained. Main advantages offered by the traction method are summarized as follows: (1) The optimal free boundary shape is obtained. (2) It is not necessary to refine the mesh. (3) It assures a smooth boundary shape. (4) It can be easily implemented combining with a standard FEM code.

In this paper the rigidity design problem of a solid structure with variable Neumann boundary is formulated. Surface tractions per unit area $\boldsymbol{P}$, body forces per unit volume $\boldsymbol{f}$ and hydrostatic pressures $p\boldsymbol{n}$ are applied on the specified regions. The compliance is minimized subjected to a volume constraint and the state equation. Using the Einstein summation convention and the partial differential notation for the spatial coordinates, the shape sensitivity function G for this problem is derived as follows:

$$G = [2 f_i v_i - 2e_{ijkl} v_{k.l} v_{i,j} + \Lambda]_{on\ \Gamma} + [2\nabla(P_i v_i)\cdot \boldsymbol{n} + 2\kappa P_i v_i]_{on\ \Gamma_1} + [2\mathrm{div}(p v_i)]_{on\ \Gamma_2} \qquad (1)$$

where $\boldsymbol{e}, \boldsymbol{v}, \Lambda, \kappa$ and $\boldsymbol{n}$ denote an elastic tensor, the displacement vevtor, the Lagrange multiplier for volume constraint, twice of the mean curvature and an outward normal vector, respectively. The notations Γ, Γ_1 and Γ_2 indicate the whole design boundaries, the boundaries surface tractions are applied and the boundaries hydrostatic pressures are applied, respectively.

The proposed method is applied to three design problems. The initial volume is set as a constraint value in all problems. The first problem is a both ends fixed beam under downward surface tractions $\boldsymbol{P}$. The initial shape and the obtained result are shown in Fig. 1-(a) and (b). Upper and lower boundaries are set as the design boundaries. For comparison with (b), the result obtained by only the first term in the right hand side of eqation (1) (i.e., strain energy density) as G is shown in Fig. 1-(c). The iteration histories for both cases are shown in Fig. 2. As we expected, the compliace of the final shape (b) is lower than that of (c). The second one is a cylindrical tower under the self-weight $\boldsymbol{f}$ and a downward surface traction $\boldsymbol{P}$. The last one is a dam under the self-weight $\boldsymbol{f}$ and hydrostatic pressure $p\boldsymbol{n}$. The initial shapes and the obtained results are shown in Fig. 3 (a),(b) and Fig. 4 (a),(b), respectively. The validity and practical utility of this method for the shape design of solid structures with variable Neumann boundaries were verified from the obtained results.



(a) Initial shape (b) Final shape (c) Final shape (by strain energy density)

Fig. 1 Calculated result of both ends fixed beam

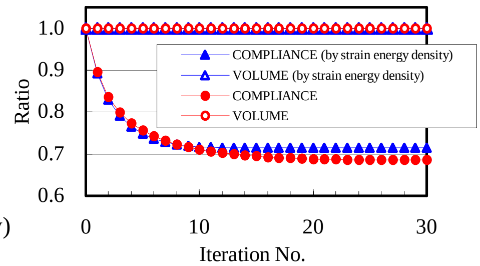
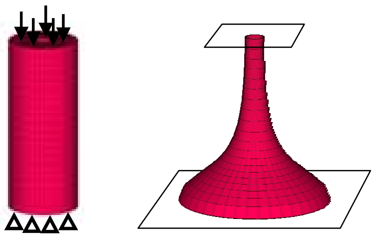
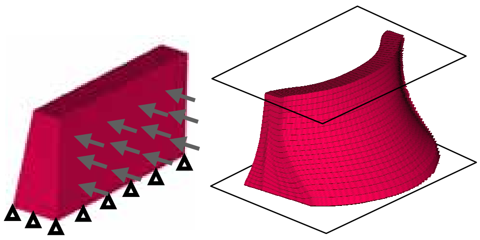


Fig. 2 Iteration histories of both ends fixed beam



(a) Initial (b) Final

Fig. 3 Calculated result of cylindrical tower



(a) Initial (b) Final

Fig. 4 Calculated result of dam

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