8th. World Congress on Computational Mechanics (WCCM8)
5th. European Congress on Computational Methods in Applied Sciences and Engineering (ECCOMAS 2008)
June 30 – July 5, 2008
Venice, Italy

A MESHLESS RBF COLLOCATION METHOD FOR POTENTIAL PROBLEMS IN NONCONVEX DOMAINS

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Key Words: *RBF Collocation Method, Elliptic Problem, Nonconvex Domains, Domain Extension.*

ABSTRACT

Recently, the multiquadric radial basis function (RBF) collocation method has been widely applied to solve partial differential equations due to its excellent capability in scattered data fitting and convergence rate of exponential order. However, in the literature, the applications are usually restricted to simple convex domains with only few exceptions [1-2]. In fact, the multiquadric RBFs can only fit scattered data defined in topologically rectangular domains [3]. Therefore, to employ the method to solve a problem in a nonconvex domain, one usually needs to incorporate some domain decomposition techniques in a typical solution procedure [4] so that the domain can be decomposed into a set of convex and topologically rectangular sub-domains. In this way, iterations in computations are required. The whole computation procedure becomes not only complicated but also time consuming.

In this paper, we introduce a fictitious domain extension approach that extends the nonconvex physical geometries defined in a problem to a topologically rectangular domain. The idea is derived from the superposition principle developed in the potential flow theory. In this approach, the definition domains of the governing equation and its solution are extended to the fictitious region and its boundary. The physical boundary conditions are still specified on the boundaries of the physical domain. And then the RBF collocation method is applied in the new extended domain.

We first applied this approach to a sudden-contraction potential flow problem. The physical domain is shown in Figure 1(a). The variable ϕ denotes the stream function. Two different domain extensions were devised as shown in Figure 1(b) and (c) which also show the collocation point distributions. The streamlines are shown in Figure 2(a). It is found that both domain extensions lead to similar results. To compare the results, we also show the streamline pattern computed from the finite difference method in Figure 2(b). It can be found that they are similar to those obtained by the present method except the part near the region of sudden contraction.

Studies of several other test problems with arbitrary geometries were also conducted. We demonstrate that the solution can be directly obtained without domain

decompositions and iterations. We also tested the present approach for problems with different types of boundary conditions. The results show that the new approach is simple, efficient and accurate.

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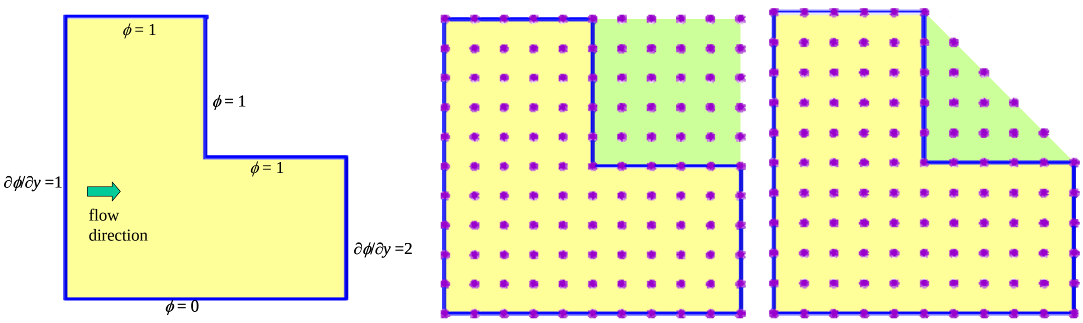
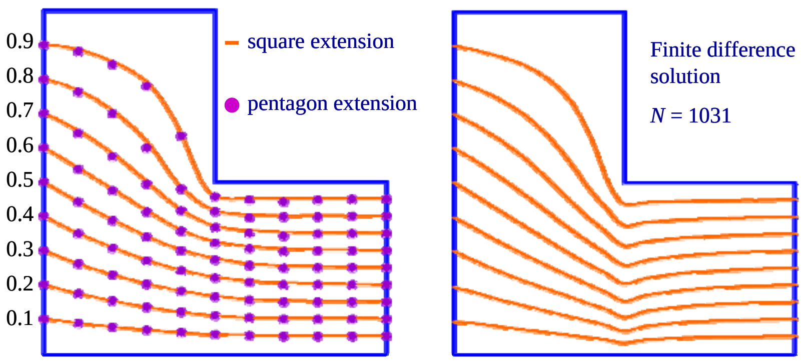


(a) physical domain. (b) domain extension 1. (c) domain extension 2.

Figure 1 Potential flow in a channel of sudden contraction.



(a) Present method. (b) Finite difference method.

Figure 2 Streamlines of the flow.