8th. World Congress on Computational Mechanics (WCCM8)
5th. European Congress on Computational Methods in Applied Sciences and Engineering (ECCOMAS 2008)
June 30 – July 5, 2008
Venice, Italy

A HYBRID METHOD FOR TRANSIENT WAVE PROPAGATION IN A MULTILAYERED SOLID

***X.Y.Su[1], J. Y.Tian[2]**

[1] Peking University
NO. 5 Yiheyuan Road, Haidian District,
Beijing, P.R.China, 100871
Email :xyswsk@pku.edu.cn

[2] Institute of Crustal Dynamics, China Earthquake Administration
Subbox 28, P.O.Box 2855,Beijing ,P.R.China, 100085
Email:j_tian@eq-icd.cn

Key Words: *Reverberation Matrix, Cagniard-De Hoop method , Generalized rays.*

ABSTRACT

Transient elastic-wave propagation in a multilayered medium plays an important role in the fields of seismology, ocean acoustics, and non-destructive evaluation. Recently, the reverberation matrix method has been presented to investigate the transient elastic waves in isotropic and transversely layered solid, which was originally developed to investigate the transient elastic waves in frames. Considering the local scattering relations at interfaces and the transfer relations in the layer, a reverberation matrix $\mathbf{R}$ is introduced to formulate the laplace-domain displacement in the multilayered solid as $\mathbf{U}(x,y,p)=\dfrac{p^2}{2\pi i}\int_{\eta_1-i\infty}^{\eta_1+i\infty}(\mathbf{A}_u\mathbf{PH}+\mathbf{D}_u)[\mathbf{I}-\mathbf{R}]^{-1}\mathbf{s}e^{p\eta x}d\eta$. The replacement of the inverse of the matrix $\mathbf{I}-\mathbf{R}$ by a power series $[\mathbf{I}+\mathbf{R}+\mathbf{R}^2+...+\mathbf{R}^N+...]$ through the Neumann expansion，the displacements can be rewritten as a series of ray group integrals $\mathbf{U}(x,y,p)=\dfrac{p^2}{2\pi i}\int_{\eta_1-i\infty}^{\eta_1+i\infty}(\mathbf{A}_u\mathbf{PH}+\mathbf{D}_u)\sum_{n=0}^{\infty}\mathbf{R}^n\mathbf{s}e^{p\eta x}d\eta$. Each ray group integral containing n^{th} power of reverberation matrix represents the set of n times reflections and the transmissions of source waves arriving at receivers in the multilayered solid. However, at present, ray group integrals are numerically evaluated by fast inverse Laplace transform (FILT). With the larger calculation burden and the lower computation precision, FILT is difficult to exactly confirm the arrival time of elastic waves so that it is impossible to truncate the ray group integrals accurately. Furthermore, with the increase of calculation time, the FILT results will trend unstable, which limits the further application of reverberation matrix method.

In order to calculate the ray group integrals efficiently, in this paper we build a clear connection between the ray group integrals and generalized-ray integrals. Extracting phase functions from reverberation matrix and receiver matrix, each of ray group integrals can be further expanded into the sum of a series of the generalized ray integrals $\mathbf{U}(x,y,p)=\sum_{n=0}^{\infty}\sum_{j=1}^{2N-2}\sum_{q+r...+v=n}\dfrac{p^2}{2\pi i}\int_{\eta_1-i\infty}^{\eta_1+i\infty}\left(\mathbf{G}_a\mathbf{s}(p)e^{-pt_1}+\mathbf{G}_d\mathbf{s}(p)e^{-pt_2}\right)d\eta$, each of which can be exactly evaluated by Cagniard-De Hoop method in the generalized ray theory theory.

The numerical example demonstrates that this hybrid method takes full advantage of automatic formulation of reflection or refraction coefficients of reverberation matrix method and high-precise calculation of generalized ray method. However, with the increase of the layer number and the times of reflection or refraction from interfaces, the number of generalized ray will increase exponentially so that this hybrid method will limit to the high-precision analysis of early-time response of the multilayered solid, which is similar with the generalized ray method.

REFERENCES

[1] J.E.White, Seismic waves: Radiation, transmission, and attenuation, McGraw-Hill, New York (1965).

[2] J.D.Achenbach**,** Wave propagation in elastic solids, North-Holland, Amsterdam (1973).

[3] Y. H. Pao and R. R. Gajewski**, "**The generalized ray theory and transient responses of layered elastic solids", in *Physics Acoustics*, **13**:183-265, Academic Press, New York(1977).

[4] X.Y.Su, J.Tian, and Y.H.Pao, **"**Application of reverberation-ray matrix to the propagation of elastic waves in a multi-layered solid", *Int. J. Solids. Struct.* , **39**: 5447- 5463(2002).

[5] J.Tian, W.X.Yang, and X.Y.Su, **"**Transient elastic waves in a transversely isotropic laminate impacted by axisymmetric load", *J. Sound.Vib*, **289**: 94-108(2006).