

## MODELING THE RHEOLOGICAL BEHAVIOUR OF ASPHALT BASED ON FRACTIONAL TIME DERIVATIVES AND NEURAL NETWORKS

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### ABSTRACT

Rheological models are widely used to mathematically describe the behavior of materials that exhibit viscous deformation components. The majority of these rheological models are based on time derivatives of integer order, e.g. NEWTON body, MAXWELL body, KELVIN body or BURGERS body. To determine the parameters of these rheological models creep tests are generally used. These tests usually consist of a loading phase followed by a phase without loading. The loading phase can be used to study the creep strain development while the phase without loading delivers information on the strain recovery characteristics. However, research has shown that it is not possible to match both phases (creep strain development and strain recovery) with only one set of parameters when time derivatives of integer order are used for the rheological formulation, see [1].

A method of solution to this problem has arisen with rheological models on the basis of time derivatives of real order, so called fractional rheological models. An application of fractional rheological elements for modeling the creep behavior of asphalt mixes is presented in this paper. In [2] it is shown that fractional differential equations exhibit special time-history characteristics. These characteristics need to be considered when incorporating boundary conditions for the solution of the fractional differential equation. This means that the development of strain is not only influenced by the current strain state and the current stress rate, but also by the entire history of stress. For an incremental solution this leads to a significant increase in computational effort, as the single stress states of all the previous increments have to be available in order to compute the strain of the current increment. The computational effort increases with the number of increments analyzed. A study on this problem will be discussed within of this paper.

To overcome this problem fractional creep functions are approximated by (a number of) DIRICHLET-series, see [3]. If DIRICHLET-series are used instead of fractional creep functions the influence of the strain history on the current strain development can be lumped into one factor per series. However, for long load duration, the DIRICHLET-

series and the fractional creep functions deviate from each other, as the DIRICHLET-series exhibit a strain limit while the fractional creep functions are unlimited in strain.

There are a number of papers that have been published more or less recently, discussing strategies for reducing the computational effort associated with the aforementioned time-history effects of the fractional differential equation, e.g. [4], [5], [6], [7] and [3]. The majority of these papers focus on the fading impact of previously imposed stresses on the current strain increment with progressing time. It is speculated that changes in stress with a certain distance in time can be neglected as they have almost no impact on the current strain increment. This might or might not lead to accurate computational results. In any case, the strategies developed in these papers require a definition (guess) of a “time horizon” that separates between ranges in time where changes in stress do have impact on the current strain increment and ranges in time with no impact. A study on this phenomenon will be carried out in this paper.

A further goal of this paper is the development of an artificial neural network that accounts for the time history effects of viscous materials (such as asphalt mixes) with no need for defining time horizons. The network can be operated without having the entire strain history available. For this reason, so called feedback loops are introduced. Architecture and functionality of the network are described within the paper. Characteristics like performance of the network and limits of application are discussed. Actual test results are used to train the network. Furthermore, the performance of the network is verified by means of a comparison between (network-) predictions and test results that have not been used for training proposes. Moreover, the computational effort associated with approaches based on fractional differential equations and the computational effort caused by the neural network is juxtaposed.

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