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ABSTRACT TITLE

Numerical Simulations of Viscous Compressible Flows in Moving Domains with High Order Discontinuous Galerkin Methods

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ABSTRACT

Problems of deformable or moving domains appear in many engineering applications including flapping flight, aeroelastics and other fluid structure interaction problems. Despite great research efforts in designing an efficient and robust numerical method for simulations of those applications involving moving computational domains, challenges remain with the handling of deformable and highly distorted grids, convergence, conservativity and accuracy. In recent years there is a growing interest in developing high order approximation methods in computational mechanics. Especially, there has been a substantial progress on the development of discontinuous Galerkin methods for a wide range of applications [1], largely due to its ability of providing highly accurate solutions on arbitrary unstructured grids as well as its convergence, stability and compactness.

In this work, a numerical method for solving compressible Navier-Stokes equations on deformable, moving domains is proposed. The method uses discontinuous Galerkin spatial discretization written in the context of ALE (arbitrary Lagrangian Eulerian) framework for deformable domains. The grid velocity is computed using the velocity smoothing [2] with variable diffusivity being obtained from mesh size functions. In addition to the ALE treatment, a simple remeshing technique for implicit geometry described by distanced functions is employed to handle the distorted grids if any. Conservation of the solutions is preserved by the ALE formulation and a conservative solution reconstruction procedure efficiently implemented in the context of DG discretization.

As an application, a simple problem of oscillating circular cylinder at low Reynolds number is carried out using the proposed approach. Motion of the cylinder is prescribed via position of the center $y_c(t) = a\sin(2\pi f t)$ where f and a are the frequency and amplitude of the oscillation. The forced frequency of the oscillation is chosen to be in the range of the vortex-shedding frequency f_s of the wake expressed as nondimensional Strouhal number, $St_s = f_s D/U_0$ where U_0 being free stream velocity and D the diameter of the cylinder. Simulation results are shown in Figures 1 and 2 for flow at $Re = 1000$, $M = 0.1$ and different Strouhal numbers of $St = 0.15, 0.21$. The method is also applied for simulations of a NACA0012 airfoil of chord $c = 1$ rotating about its leading edge as $\alpha = \alpha_{max}\sin(2\pi f t)$ with frequency of $f = 0.2$ and the angular amplitude of $\alpha_{max} = \pi/4$ under the freestream flow of Mach number $M = 0.2$ and Reynolds number $Re = 1000$ as shown in Figures 3 and **??**.

The proposed approach of ALE DG method has shown a promising performance for moving boundary applications. Moving grids are efficiently handled using a simple velocity smoothing algorithm; the quality is controlled and well preserved by a fast grid regeneration technique. The advantage of providing high order approximation on unstructured grids has enabled the use of the proposed approach for problems of complex geometry with coarser meshes. In addition to problems with moving domains, the method is also suitable for simulations of fluid-structure interaction applications where the movement of the boundaries are provided from a structure dynamics solver. Research on this area is in progress.

REFERENCES

[1] B. Cockburn and C.-W. Shu, "Runge-Kutta Discontinuous Galerkin Methods for Covective-Dominated Problems", Review Article, *J. Sci. Comp.*, Vol. **16**, 173–261, 2001.

[2] R. Lohner and C. Yang, "Improved ALE Mesh Velocities for Moving Bodies", *Comm. Num. Meth. Engin.*, Vol. **12**, 599-608, 1996.

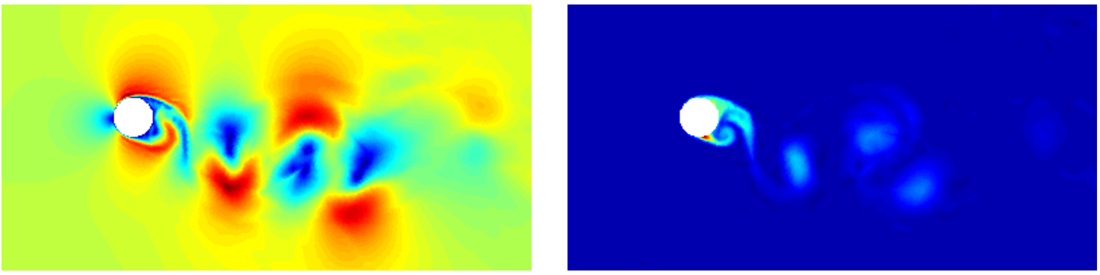

Figure 1: Instantaneous profiles of Mach number (right) and entropy (left) for flow over a vertically oscillating cylinder at $Re = 1000$, $M = 0.1$; Strouhal number $St = 0.15$ and amplitude $a = 0.3$ using DG p=3

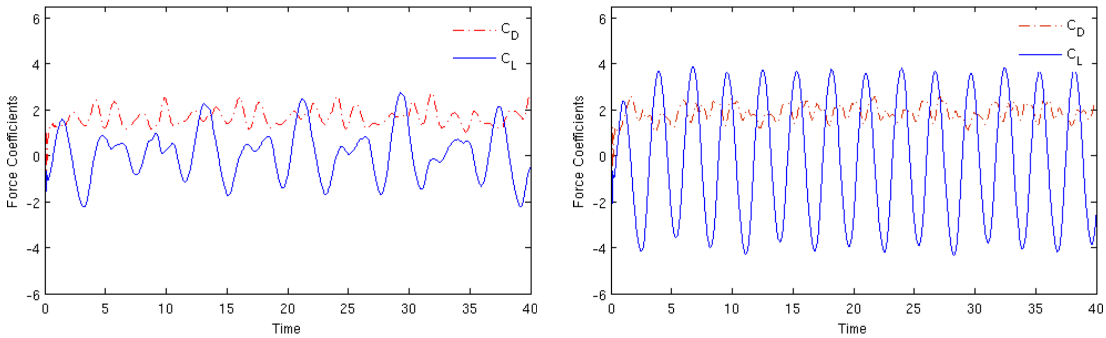

Figure 2: Lift and drag coefficients for an oscillating cylinder at $Re = 1000$, $M = 0.1$, amplitude $a = 0.3$; for non-lock-in case at Strouhal number $St = 0.15$ (left) and lock-in case at $St = 0.21$ (right). Simulation was done using ALE DG discretization with approximating polynomial order p=3.

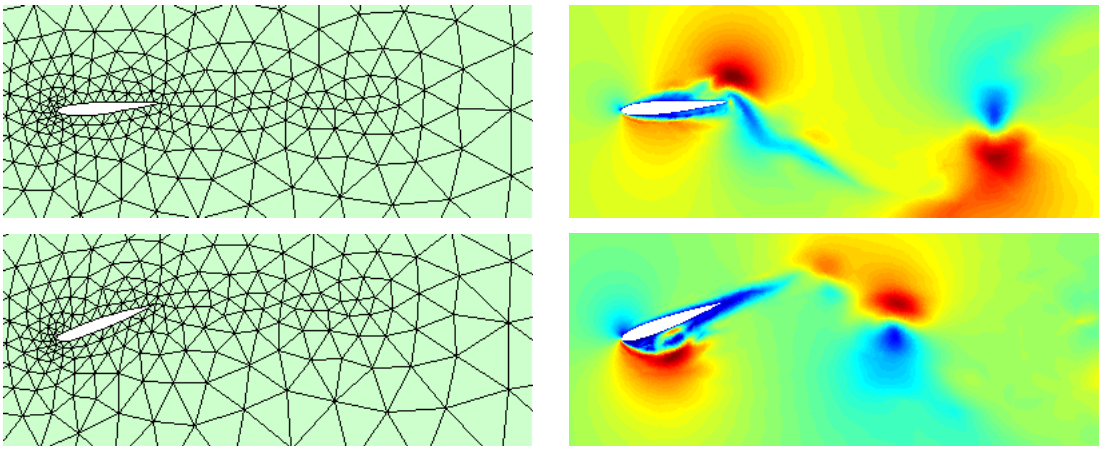

Figure 3: Grids (left) and instantaneous Mach number profiles (right) at different times for a flapping airfoil with frequency $f = 0.2$ and angular amplitude $\alpha_{max} = \pi/4$ under the flow of $Re = 1000$, $M = 0.2$. Note at how the grid has moved and been preserved the quality during the flapping of the airfoil.