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Flux-free Finite Element Method for Immiscible Two-fluid Flows

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ABSTRACT

For last two decades a lot of attentions has been paid to the numerical analysis for flow problems with the moving surface or interface. A main concern in these studies is the numerical reproduction of the evolution of free surface or interface, however, we have also to take care of the mass preserving of fluids. In immiscible incompressible two-fluid flows, the mass of each fluid should be preserved due to the incompressibility and immiscibility, however, we have often experienced the *gain or loss* of the mass of both fluids in the numerical simulations for these problems. Recently we have proposed the flux-free finite element method [3] based on the Eulerian framework for immiscible incompressible two-fluid flows, which is defined so as to preserve the mass of each fluid. Introducing the flux functional [1] on the interface, our method is derived from the mixed variational formulation including the flux-free constraint using the Lagrange multiplier technique.

In this study we are concerned with some mathematical properties of our method such as basic error estimates and convergence [2]. In general, the mathematical model for immiscible incompressible two-fluid flows is described by the Navier-Stokes equations with discontinuous density and viscosity, that is, Navier-Stokes interface problems. In this study we restrict ourselves to the generalized Stokes interface problems for simplicity. Since there are few results concerning the regularity of the solution for the Stokes interface problems, we discuss some convergence properties of our method by referring some results for the elliptic interface problems.

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