

## XFEM AND MESH ADAPTATION: A MARRIAGE OF CONVENIENCE

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### ABSTRACT

In this work, we present an odd but convenient coupling of the extended finite element method (XFEM), a method developed with the intention of simulating the crack growth without remeshing [1], with a mesh adaptation strategy based on error estimates.

The basic principle of XFEM is to take into account known features of the solution of a problem in the approximation of the unknown field. In fracture mechanics, the displacement field is enriched with

- a discontinuous function across the crack surface, which allows the simulation of crack growth in a finite element framework without the need to build a conforming mesh at each step of the propagation,
- functions with singular derivative near the crack front that span the near-tip expansion of the displacement field, which improve the accuracy of the method.

Even though the mesh does not need to match the crack surface, in practice, the size of its elements must be small enough with respect to characteristic lengths like the crack length and the crack radius of curvature. In our experience on real-world crack propagation simulations in aeronautical structures, the mesh available from the structural analysis with the FEM of a safe component is not appropriate for the damage-tolerance analysis of this cracked component with the XFEM.

For this reason, a mesh refinement procedure tailored to the XFEM is desirable. A simple approach is to divide recursively the elements that are close to the crack until their size is deemed small enough. A more sound approach is to refine the mesh on the basis of an error estimation.

Two a-posteriori, recovery-based error estimation methods were proposed for the XFEM in linear elastic fracture mechanics by the authors recently [2,3]. Both methods are based on a procedure that incorporates the known behaviour of the solution in order to build a recovered derivative field from the raw XFEM solution. The recovered field is smoother than the raw file and closer to the exact solution. The error on the numerical solution is then estimated by the norm of the difference between the raw and smooth strain fields, like in the Zienkiewicz-Zhu a-posteriori error estimation method in the FEM.

From these local a-posteriori error estimation methods, three types of adaptivity may be considered:

- h-adaptivity, where the mesh is refined;
- p-adaptivity, where the order of the regular shape functions and those multiplying the enrichments is increased;
- e-adaptivity, where the size of the enriched region and the nature of the enrichment are modified.

This presentation investigates the first of these paths, mesh adaptation in an XFEM framework. The method is first presented on simple problems with a reference solution and then applied to a crack propagating in a blade of an engine stator.

## REFERENCES

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