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Computable error bounds for elliptic optimal control problems

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ABSTRACT

We consider the following optimal control problem:

$$\min \frac{1}{2}\int_\Omega (y(x)-y_d(x))^2 dx + \frac{\gamma}{2}\int_\Omega u(x)^2 dx$$

subject to

$$\begin{aligned} -\Delta y + d(y) &= u \qquad \text{in } \Omega, \\ y &= 0 \qquad \text{on } \partial\Omega, \end{aligned}$$

and

$$u_a(x) \le u(x) \le u_b(x) \qquad \text{a.e. on } \Omega.$$

Let us asssume that a local solution $\bar u$ with associated $\bar y$ exists. To compute an approximate solution $\bar u_h$ to $\bar u$ the problem has to be discretized. A-priori error estimates of $\|\bar u - \bar u_h\|$ and $\|\bar y - \bar y_h\|$ have been developed in the recent past. They rely heavily on a convexity assumption on the unknown solution of the original problem. Moreover, a-posterior estimates are available, which can be used in mesh adaptation strategies. In the talk, we propose a method to compute upper bounds of $\|\bar u - \bar u_h\|_{L^2(\Omega)}$ and $\|\bar y - \bar y_h\|_{L^2(\Omega)}$. That is, we will prove estimates of the errors that only depend on quantities that are available after the discretized problem had been solved. Moreover, these estimates are potentially applicable for mesh refinement.