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AN ENERGY-PRESERVING DISCRETE ELEMENT METHOD FOR LINEAR ELASTICITY IN LARGE DISPLACEMENT

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ABSTRACT

Since their first use by Hoover *et al* [3] in models for crystalline materials and Cundall & Strack [1] in geotechnical problems, Discrete Elements methods (DEM) have found a large field of application in granular materials, soil and rock mechanics. The handling of a set of particles interacting by means of forces and torques can lead to a wide variety of macroscopic models for the material at hand, depending on the different small-scale models chosen for the interaction forces and torques.

In the software Mka3D (Mariotti [4]), we have been able to simulate the behaviour of a three-dimensional linear elastic material through a particular choice for the expression of the forces and torques between particles. In small displacements, using a cartesian partition of the volume into cubic particles, the model yields a second-order spatial discretization of the equations of linear elastodynamics :

$$\rho\underline{\ddot{X}} \quad = \quad \frac{E}{2(1+\nu)}\Delta\underline{X} + \frac{E}{2(1+\nu)(1-2\nu)}\underline{\mathrm{grad}}(\mathrm{div}\underline{X})$$

This allows seismic simulations using only 10 particles per wavelength (Mariotti [4]).

However, the use of the model is not restricted to small displacements. Another important property of the forces and torques used is that they derive from a discrete potential energy, reflecting the hamiltonian structure of the continuum equations onto the discretized model. Therefore, large displacements can be considered. We preserve this hamiltonian structure at the discrete level in time by using a symplectic algorithm. We implemented second-order Velocity-Verlet scheme for translations, and for rotations, the RATTLE second-order scheme preserving the hamiltonian structure of the rigid-body movement described in Hairer, Lubich, Wanner [2]. As the expressions of forces and torques only depend on the relative positions and orientations of the particles, we are therefore able to follow the dynamics of large displacements, while ensuring a good preservation of energy over long-time simulations. We have passed such energy-conserving tests in the case of an oscillating beam (with a three-dimensional

movement) and of a pinched oscillating cylinder, over at least 100 periods (between 100000 and one million time steps).

Adding damping to the model, we can compute static elastic displacements under load once the system has settled down and the kinetic energy has vanished. Figure 1 shows the elastic displacement of a pinched cylinder computed using Mka3D.

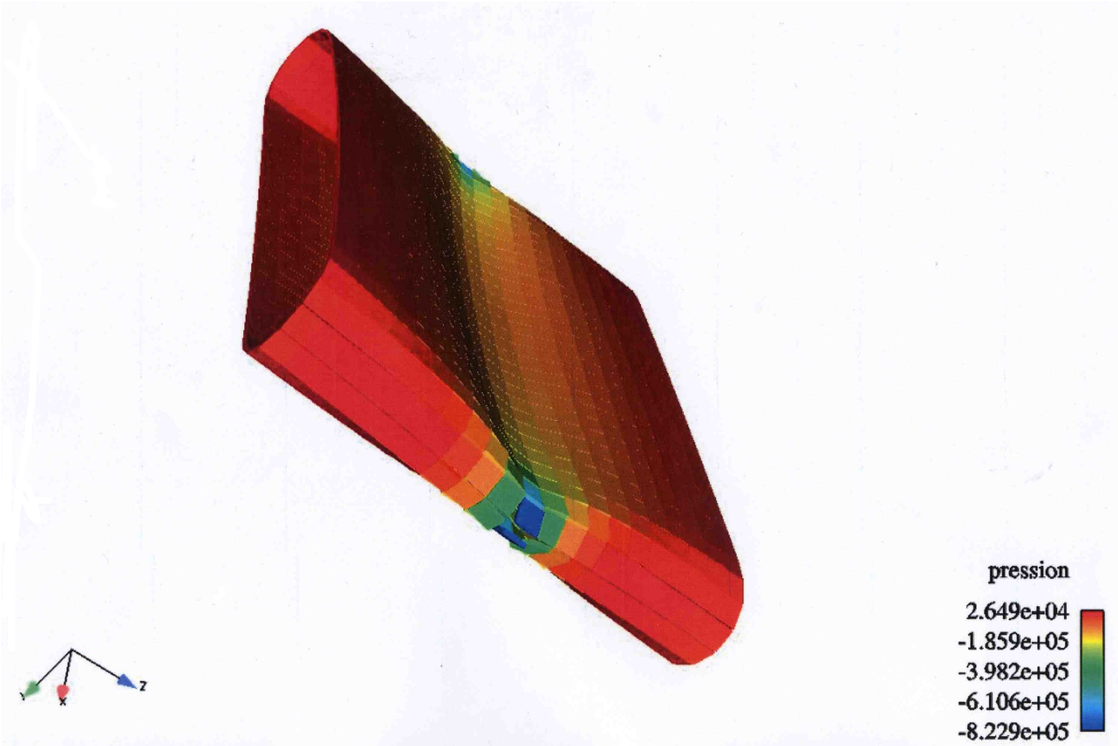

Figure 1: Test of the pinched cylinder with Mka3D

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