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COMPARISON OF TWO FULL-WAVE MODELS FOR A LOUDSPEAKER

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ABSTRACT

The numerical modeling of wave propagation poses a significant challenge in scientific computation. Historically, several approaches have been explored in the quest for a stable method that can efficiently resolve wave without excessive numerical dissipation or dispersion. Two promising candidates for solving the wave propagation problems with a reduced computational cost are studied in this paper. These methods are the discontinuous Galerkin (DG) [1] for the time domain solutions and the ultra weak variational formulation (UWVF) [2] for the solutions that are computed in the frequency domain.

As the mathematical model we assume that Ω is a bounded Lipschitz domain in $\mathbb{R}^3$, Γ is its boundary, $\boldsymbol{x} = (x_1, x_2, x_3) \in \Omega$ is the spatial variable and $t \in [0, T]$ time. Then, the linear acoustic wave equation with the boundary condition on the boundary Γ and initial condition with $t = t_0$ can be written as

$$\frac{1}{c^2\rho}\frac{\partial^2 u}{\partial t^2} = \nabla \cdot \left(\frac{1}{\rho}\nabla u\right) + f \quad \text{in } \Omega, \tag{1}$$

$$\frac{\partial u}{\partial t} = u_0 \quad \text{at } t = t_0, \tag{2}$$

$$u = u_1 \quad \text{at } t = t_0, \tag{3}$$

$$\sigma\frac{\partial u}{\partial t} + n \cdot \left(\frac{1}{\rho}\nabla u\right) = Q\left(-\sigma\frac{\partial u}{\partial t} + n \cdot \left(\frac{1}{\rho}\nabla u\right)\right) + \sqrt{2\sigma}g \quad \text{on } \Gamma, \tag{4}$$

where u is the acoustic pressure, c is the wave speed, ρ is the density, and f is a source term. In Eqs. (2) and (3) the u_0 and u_1 contain the given initial values and in Eq. (4) n is a spatial outward unit normal on Γ, g is a source function, and σ is a real and positive parameter. The Helmholtz problem is obtained by setting the acoustic pressure field $u(\boldsymbol{x}, t) = \hat{u}(\boldsymbol{x})e^{-i\omega t}$.

For the DG and UWVF, the domain Ω is partitioned into a collection of finite elements (tetrahedra are a natural choice in 3D). After the mesh partitioning, the weak formulation can be written individually for each element. Furthermore, the communication between adjacent elements is handled using a numerical flux. The final weak form for the problem is obtained by summing over all of the elements. The more detailed derivation for the UWVF method can be found from [3] and for the DG method from [4].

Several acoustical phenomena can be simulated using this model. The main goal of this study is to investigate the frequency response of the loudspeaker on the acoustic axis. In addition, directivity patterns can also be obtained from the computations. These modeled results are also compared to experimental measurements. Figure 1 shows an example of the volume mesh and the surface mesh of the used in the simulations.

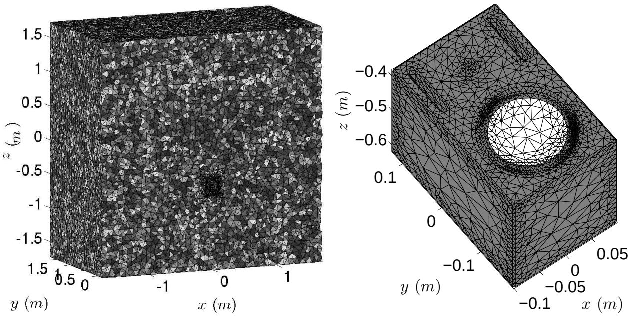

Figure 1: An example of the element partition (left) and the surface mesh of the loudspeaker (right).

Figure 2 illustrates the full-wave solution computed using the mesh shown in Figure 1. These results are obtained using frequency $f = 1000$ Hz with the UWVF method.

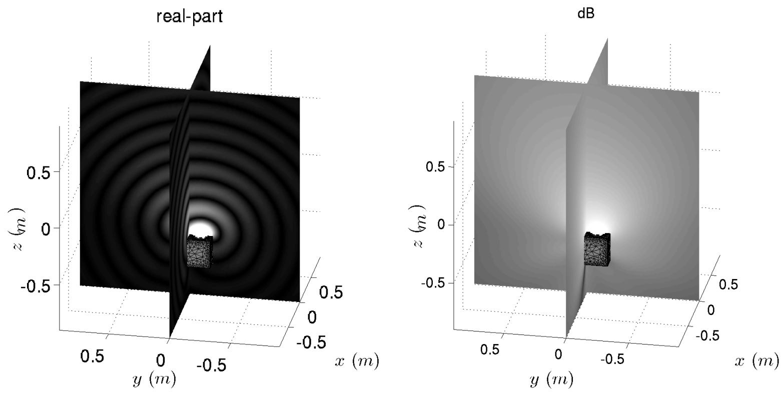

Figure 2: An example of the pressure fields (real-part and sound pressure level in decibel scale (dB)).

As a conclusion of the results, accurate solutions are obtained using the UWVF and DG methods for numerically simulating loudspeaker related problems.

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