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QUASI-GASDYNAMIC NUMERICAL ALGORITHM FOR UNSTEADY FLOW SIMULATIONS

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ABSTRACT

Numerical algorithm for calculation viscous compressible gasdynamic flows is presented. It is based on a special form of regularization in Navier-Stokes equation system that includes additional dissipative terms proportional to a small parameter in τ for all equations of the system. This new equations were called quasi-gas-dynamic (QGD) ones. The first variant of this system was presented in [1] and developed later in, e.g. [2] – [5]. The first gasdynamic systems with nonclassical continuity equation are presented in [6] and [7]. Others models can be found in, e.g. [8] -[11]. In the form of conservation laws QGD system writes

$$\frac{\partial \rho}{\partial t} + div\vec{j}_m = 0, \tag{1}$$

$$\frac{\partial (\rho\vec{u})}{\partial t} + div(\vec{j}_m \otimes \vec{u}) + \vec{\nabla} p = div\Pi, \tag{2}$$

$$\frac{\partial}{\partial t}\Big[\rho\Big(\frac{\vec{u}^2}{2}+\varepsilon\Big)\Big] + div\Big[\vec{j}_m\Big(\frac{\vec{u}^2}{2}+\varepsilon+\frac{p}{\rho}\Big)\Big] + div\vec{q} = div(\Pi\cdot\vec{u}), \tag{3}$$

where the closing relations for mass flux vector, shear stress tensor and heat flux vector are

$$\vec{j}_m = \rho(\vec{u}-\vec{w}), \quad where \quad \vec{w} = \frac{\tau}{\rho}[div(\rho\vec{u}\otimes\vec{u}) + \vec{\nabla} p], \tag{4}$$

$$\Pi = \Pi_{NS} + \tau\vec{u}\otimes\Big[\rho(\vec{u}\cdot\vec{\nabla})\vec{u} + \vec{\nabla} p\Big] + \tau I\Big[(\vec{u}\cdot\vec{\nabla})p + \gamma p div\vec{u}\Big], \tag{5}$$

$$\vec{q} = \vec{q}_{NS} - \tau\rho\vec{u}\Big[(\vec{u}\cdot\vec{\nabla})\varepsilon + p(\vec{u}\cdot\vec{\nabla})\Big(\frac{1}{\rho}\Big)\Big]. \tag{6}$$

Here Π_{NS} and $\vec{q}_{NS}$ are the Navier-Stokes shear- stress tensor and heat flux vector, respectively, τ is a small parameter, that has a dimension of time. The system (1) – (6) is completed by the state equations for a perfect gas and expressions for coefficients of viscosity, heat conductivity and τ.

For perfect gas the entropy production for QGD system is the entropy production for Navier-Stokes system completed by the additional terms in τ, that are the squared left-hand sides of classical stationary Euler equations with positive coefficients:

$$X = \text{æ}\Big(\frac{\vec{\nabla}T}{T}\Big)^2 + \frac{(\Pi_{NS} : \Pi_{NS})}{2\eta T} + \frac{p\tau}{\rho^2 T}\Big[div(\rho\vec{u})\Big]^2 +$$
$$+\frac{\tau}{\rho T}\Big[\rho(\vec{u}\cdot\vec{\nabla})\vec{u} + \vec{\nabla}p\Big]^2 + \frac{\tau}{\rho\varepsilon T}\Big[\rho(\vec{u}\cdot\vec{\nabla})\varepsilon + p div\vec{u}\Big]^2.$$

Above equation proves a dissipative nature of the additional τ-terms.

QGD and Navier-Stokes systems differ by the second space derivative terms of the $O(\tau)$ order. For stationary flows the dissipative terms (terms in τ) in the QGD equations have the asymptotic order of $O(\tau^2)$ for $\tau \to 0$. In a boundary layer limit both QGD and Navier-Stokes equations reduce to Prandtl system.

Terms in τ allow to construct a family of novel efficient numerical algorithms for simulations of nonstationary supersonic and subsonic gasdynamic flows. In the full paper the examples of QGD algorithms and numerical calculations for 1D, 2D and 3D problems will be presented. The possibilities in direct numerical simulations of turbulent flows will be discussed.

References

[1] T.G. Elizarova, B.N. Chetverushkin. "Kinetic algorithms for calculating gasdynamic flows." *Comput. Mathem. and Mathem. Phys.*, Vol **25**, 164–169, 1985.

[2] Yu.V. Sheretov. *Mathematical modeling of liquid and gas flows based on quasi-hydrodynamic and quasi-gasdynamic equations*. Tver State university, Tver, 2000 (in russian)

[3] T.G. Elizarova. *Quasi-gasdynamic equations and numerical methods for viscous flow computations*. Scientific world, Moscow, 2007 (in russian).

[4] T.G. Elizarova, Yu.V. Sheretov. "Theoretical and numerical investigation of quasi-gasdynamic and quasi-hydrodynamic equations." *Comput. Mathem. and Mathem. Phys.*, Vol. **41**, 219–234, 2001.

[5] T.G. Elizarova, M.E. Sokolova, Yu.V. Sheretov. "Quasi-gasdynamic equations and numerical simulation of viscous gas flows." *Comput. Mathema. and Mathem. Phys.*, Vol **45**, 524 - 556, 2005.

[6] N.A. Slezkin. "About a differential equations of a gas motion." *Dokl. of USSR Ac. of Science*, Vol. **77**, 205–208, 1951.

[7] S.V. Vallander. "Equations of viscose gas motion." *Dokl. of USSR Ac. of Science*, Vol. **58**, 25–27, 1951.

[8] Yu.L. Klimontovich. "About the necessity and possibility of unified description of kinetical and hydrodynamical processes." *Theoretical and Mathematical physics*, Vol. **92**, 312-330, 1992.

[9] B.V. Alexeev. *Generalized Boltzmann physical kinetics*. Elsevier, Amsterdam, 2004.

[10] H. Brenner. "Navier–Stokes revisited." *Physica A.* Vol. **349**, 60–132, 2005.

[11] H.C. Ottinger. "Beyond equilibrium thermodynamics." Hoboken: John Wiley, 2005.